PCT

WORLD INTELLECTUAL PROPERTY ORGANIZATION
International Bureau

INTERNATIONAL APPLICATION PUBLISHED UNDER THE PATENT COOPERATION TREATY (PCT)

(51) International Patent Classification 5 : G06F 15/30

A1

(11) International Publication Number: WO 92/15064

(43) International Publication Date: 3 September 1992 (03.09.92)

(21) International Application Number: PCT/US92/01445

(22) International Filing Date: 24 February 1992 (24.02.92)

(30) Priority data: 660,671 25 February 1991 (25.02.91) US

(71) Applicant: THE PRUDENTIAL INSURANCE COMPANY OF AMERICA [US/US]; 751 Broad Street, Newark, NJ 07102 (US).

(72) Inventors: FONG, H., Gifford ; 1202 Franklin Lane, Lafayette, CA 94549 (US). VASIVECK, Oldrich, A. ; 3188 Kingsley Place, Lafayette, CA 94549 (US).

(74) Agents: HALVORSON, David, R. et al.; Blakely, Sokoloff, Taylor & Zafman, 12400 Wilshire Boulevard, 7th Floor, Los Angeles, CA 90025 (US).

(81) Designated States: AT, AT (European patent), AU, BB, BE (European patent), BF (OAPI patent), BG, BJ (OAPI patent), BR, CA, CF (OAPI patent), CG (OAPI patent), CH, CH (European patent), CI (OAPI patent), CM (OAPI patent), DE, DE (European patent), DK, DK (European patent), ES, ES (European patent), FI, FR (European patent), GA (OAPI patent), GB, GB (European patent), GN (OAPI patent), GR (European patent), HU, IT (European patent), JP, KP, KR, LK, LU, LU (European patent), MC (European patent), MG, ML (OAPI patent), MR (OAPI patent), MW, NL, NL (European patent), NO, RO, RU, SD, SE, SE (European patent), SN (OAPI patent), TD (OAPI patent), TG (OAPI patent).

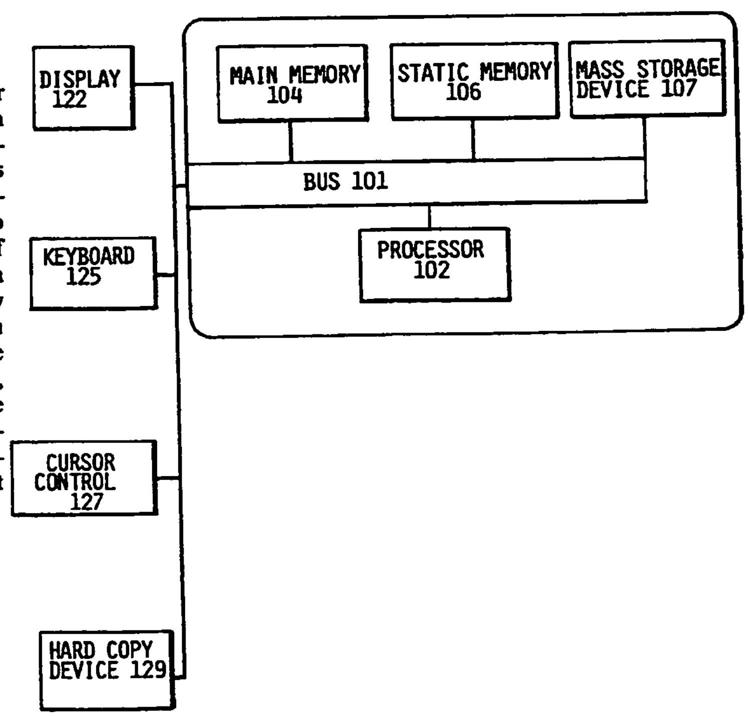
Published

With international search report.
Before the expiration of the time limit for amending the claims and to be republished in the event of the receipt of amendments.

(54) Title: METHOD AND APPARATUS FOR VOLATILITY ANALYSIS OF INTEREST RATE SENSITIVE FINANCIAL INSTRUMENTS

(57) Abstract

A financial instrument evaluation system for providing the ability to measure the exposure of a financial instrument, or portfolio of financial instruments, to changes in interest rate volatility. It is recognized by the present that the interest rate sensitive financial instruments react in valuation to both the level of interest rates and the volatility of those interest rates. Therefore, what is provided is a method and apparatus for determining a volatility exposure value indicative of the expected change in valuation of a financial instrument for each one point change in level of volatility and, further, method and apparatus for applying interest rate (level and volatility) forecasts to provide an assessment of the expected percentage change in valuation of a financial instrument given a one unit change in interest rates and/or volatility level.



FOR THE PURPOSES OF INFORMATION ONLY

Codes used to identify States party to the PCT on the front pages of pamphlets publishing international applications under the PCT.

AT	Austria	FI	Finland	ML	Mali
AU	Australia	FR	France	MN	Mongolia
BB	Barbados	GA	Gabon	MR	Mauritania
BE	Belgium	GB	United Kingdom	MW	Malawi
BF	Burkina Faso	GN	Guinea	NL	Netherlands
BG	Bulgaria	GR	Greece	NO	Norway
BJ	Benin	HU	Hungary	PL	Poland
BR	Brazil	IE	Ireland	RO	Romania
CA	Canada	IT	Italy	RU	Russian Federation
CF	Central African Republic	JP	Japan	SD	Sudan
CG	Congo	KP	Democratic People's Republic of Korea	SE	Sweden
CH	Switzerland	KR	Republic of Korea	SN	Senegal
CI	Côte d'Ivoire	LI	Liechtenstein	SU	Soviet Union
CM	Cameroon	LK	Sri Lanka	TD	Chad
CS	Czechoslovakia	LU	Luxembourg	TG	Togo
DE	Germany	MC	Monaco	US	United States of America
DK	Denmark	MG	Madagascar		
ES	Spain				

METHOD AND APPARATUS FOR VOLATILITY ANALYSIS OF INTEREST RATE SENSITIVE FINANCIAL INSTRUMENTS

1. FIELD OF THE INVENTION

This present invention relates to a method for assessing the exposure of a financial instrument (or portfolio of financial instruments) to risks and, more specifically, of assessing the exposure of a fixed income instrument (or portfolio of instruments) to risks of changing interest rate volatility.

2. BACKGROUND OF THE INVENTION

Fixed income investors have long recognized the movement of interest rates as the primary source of returns, and of risk. Indeed, interest rate changes greatly affect the value of a fixed income instrument, such as a bond; however, such rate changes have not been predictable with certainty.

In making decisions regarding purchasing, selling or holding a particular instrument—or, in the case of a portfolio, in assessing the risks and returns of the portfolio—an investor or portfolio manager who is willing to take a position with respect to the expected rate movement will structure investment(s) to benefit most from an anticipated favorable change in rates, or to lose the least in the event the expected change is unfavorable.

Other, more passive, investors may attempt to avoid taking positions (or bets) on the direction of rate changes and will rather minimize the risk of negative performance against an investment benchmark. The benchmark may be, for example, the present value of the liabilities in a pension plan funding, the value of guaranteed income certificates issued by an insurance company, or the return on a broad market index used to judge a portfolio manager's performance. No matter what the particular benchmark, the passive investor will set up investments to react to interest rate changes in the same way in which the benchmark reacts to those changes.

In either event—active exploitation of interest rate forecasting or passively guarding against interest rate movements affecting the relative valuation of an investment portfolio—the investor requires the ability to measure or assess the exposure of a portfolio to interest rate changes.

Similarly, a portfolio manager considering an investment in a single security may wish to analyze the proposed investment against other potential investments to determine which of the several investments pose the greatest risk or offer the greatest potential for return when exposed to interest rate changes.

On this note, it should be said that being aware that movement of interest rates is the primary source of returns and risks with respect to fixed income investments, quantitative measures of such returns and risks have previously been developed and are available in the prior art. Perhaps the most widely accepted tool for assessing risks / returns on fixed income investments due to interest rates fluctuations is the "duration" of the investment. Duration may be thought of as a method for comparing interest rate risk between investments with differing coupon rates and maturities. The output of the duration calculation is the weighted average of all cash flows for a particular instrument (or portfolio of instruments). Instruments with higher durations offer greater risk than instruments with relatively lower durations with respect to interest rate fluctuations (i.e., the price of an instrument with a high duration will be more greatly affected by a one point change in interest rate than the price of an instrument with a low duration).

Duration is perhaps better explained with reference to Concepts and Applications: Understanding Duration and Convexity, Chicago Board of Trade, 1990 available from the Literature Services Department, Chicago Board of Trade, 141 W. Jackson Boulevard, Chicago, Illinois (hereinafter "Concepts and Applications").

Although it is accepted that duration is one risk measure to be considered in assessing the valuation of interest rate dependent instruments, it is recognized that at least one other risk measure known as "convexity" also plays a role in interest rate exposure. Convexity may be thought of as the measure of the change of duration as interest rate moves—it is, therefore, a secondary, derivative measure.

Convexity is also explained in further detail with reference to Concepts and Applications.

It has been recognized that both duration and convexity are to be considered in assessing the risks and potential returns of an interest rate sensitive instrument; however, the present invention further recognizes that duration and convexity, taken alone, do not fully allow for assessment such risks and returns. Rather, it is recognized that some other risk measure(s) exist which contribute to assessment of these risks and returns.

Fabozzi and Garlicki, Advances in Bond Analysis & Portfolio Strategies, Probus Publishing Company, 1987 (hereinafter Fabozzi et al. further describes techniques of bond and portfolio management including discussion of duration and convexity and discussion of utilizing these tools in various portfolio and bond management applications including, by way of example, asset allocation, liability funding, analysis of fixed income securities with contingent cash flows, analysis and valuation of callable bonds, indexing of fixed income portfolios, and hedging.

One definition of convexity given by Fabozzi et al. is "that portion of a bond's price change (as yields change) that is not explained by the bond's duration" (at page 23). Although it is true that convexity may be used to explain portions of a financial instrument's price change not explained by duration, it has been discovered by the present invention that at least one additional risk measure should be considered in assessing the risks and potential rewards of a financial instrument. It should be noted that a more classical definition of convexity is given as the rate at which duration itself will change when interest rates change. Thus, the definition of convexity is classically tied to the level of interest rates and is typically not defined to describe collectively all risk measures not explained by a financial instrument's duration, as may be implied by Fabozzi et al.

Fabozzi et al., at pages 19-32, discusses the effects of interest rate fluctuations and the volatility of the bond market as an influence on option valuation. Fabozzi et al. characterizes the volatility of bond prices as a primary reason for the difference between the performance of callable and non-callable bonds. However, in discussing the volatility of bond prices, Fabozzi et al. appears to be refering to the fact that prices change; not to measuring and considering the degree of

change. In fact, Fabozzi et al. appears to recognize a tie between price changes and changes in levels of interest rates, stating at page 30:

> "The choices are difficult. Ultimately, correct macroeconomic forecasts will dominate the active/passive choice. Will volatility increase or diminish and when? Will rates go up or down and when? What is it that influences volatility? *How do interest rate changes and volatility changes trade off?* When does the volatility/interest rate forecast favor one index over the other? *These are the tough questions that should be addressed.*" (emphasis added).

Thus, Fabozzi et al. recognizes that bond prices are volatile; however, known techniques for assessing risks and potential rewards associated with financial instruments have realized that the prices of financial instruments change over time (i.e., prices are volatile); and even that interest rates themselves change over time (i.e., interest rates are volatile). However, having recognized the price of financial instruments change over time (and the level of interest rates change over time), known methods of assessing risks and evaluating potential rewards in financial instrument investments have only considered the level of interest rates at the beginning of a period and the level of interest rates at the end of a period in assessing the risks and rewards associated with such financial instruments. Such known methods have not considered the effect of volality of interest rates during the measurement period as a factor in assessing risks and rewards.

As will be seen, it is discovered by the present invention that a key risk measure in assessing valuation of bond (and similar financial instrument) prices, is the stochastic nature of volatility of interest rates, in addition to the well-established risk associated with the level of such interest rates.

Thus, it is an objective of the present invention to develop an improved method and apparatus for assessing the risk / return of an investment (or portfolio of investments).

It is a second object of the present invention to define an additional measure for assessment of risks and returns for interest rate sensitive financial instruments. In particular, the present

invention defines an additional measure for assessing risks and returns due to changing interest rate volatility.

It is yet a third object of the present invention to provide facility for a portfolio or investment manager or the like to assess the risks and potential returns of a portfolio or investment.

These and other objects of the present invention will be better understood with reference to the below Detailed Description of the Preferred Embodiment and with further reference to the several Figures.

SUMMARY OF THE DISCLOSURE

A system for analysis of the potential interest rate sensitive risks and returns of financial instruments is disclosed. The present invention recognizes that conventional methods of assessment of potential rewards and risks of interest rate sensitive financial instruments relying on expectations of changes in the level of interest rates is incomplete. Therefore, the present invention has identified an additional risk measure to be considered in assessment of interest rate related risks and rewards; specifically, it has been discovered that volatility of interest rates, in addition to the level of interest rates, affects valuation of certain financial instruments—in other words, even if the level of interest rates is identical at time T1 and time T2, the valuation of certain financial instruments may be effected by the volatility in interest rate levels between time T1 and T2.

In particular, a method for assessing the exposure of a financial instrument to interest rate changes has been defined wherein a volatility exposure value is calculated representative of the percentage change in the price of the financial instrument resulting from a one unit change in the volatility of interest rates.

Initially, the present invention calls for assessing the risks associated with volatility of interest rates, i.e., calculating the volatility exposure value. Then, in certain applications (hereinafter termed expectational applications), such as active management, an expectation as to the volatility of interest rates during a holding period is formed. The expectation and volatility exposure values are used to determine and maximize potential rewards associated with a financial instrument during the holding period for which the expectation has been formed. In other applications, (hereinafter termed non-expectational applications), such as indexing, hedging and immunization, decisions may be made regarding particular financial instruments and portfolio fomation after having calculated the volatility exposure value.

In the preferred embodiment of the present invention, the volatility of interest rates is applied as a stochastic factor in formulation of the relative risks and returns of the financial instrument.

The present invention further discloses means for calculating a volatility exposure value and means for applying an expectation of interest rate volality during a holding period and calculated volatility exposure value to a financial instrument to assess the risks and potential rewards of the financial instrument.

These and other aspects of the present invention will be described in greater detail with reference to the Detailed Description of the Preferred Embodiment and the accompanying figures.

BRIEF DESCRIPTION OF THE DRAWINGS

Figure 1 is an overall block diagram of a computer system as may be utilized to by the present invention.

Figures 2(a) and 2(b) is useful for illustrating the concept of duration.

Figure 3 is useful for illustrating the concept of convexity.

Figure 4 is an overall flow diagram of a volatility management process as may be utilized by the present invention.

Figure 5 is a flow diagram illustrating development of a model of the term structure of interest rates, as may be utilized by the present invention.

Figure 6 is a flow diagram illustrating development of computational procedures as may be utilized by the present invention.

Figure 7 is a flow diagram illustrative of application of expected volatility and levels of interest rates to an assessment of the relative risks and potential rewards associated with a financial instrument.

Figure 8 is an overall flow diagram of the current implementation of the present invention.

Figure 9 is a flow diagram illustrating input of specifications, objectives and parameters, as may be accomplished by the current implementation of the present invention.

Figure 10 is a flow diagram illustrating steps for computing an implied market volatility as may be accomplished by the current implementation of the present invention.

Figure 11 is a flow diagram illustrating a method for specifying information about an asset as may be accomplished by the present invention.

Figure 12(a) is a flow diagram illustrating a first method for specifying information about a liability as may be accomplished by the present invention.

Figure 12(b) is a flow diagram illustrating a second method for specifying information about a liability as may be accomplished by the present invention.

For ease of reference, it might be pointed out that reference numerals in all of the accompanying drawings typically are in the form "drawing number/" followed by two digits, xx; for example, reference numerals on Figure 1 may be numbered 1xx; on Figure 7, reference numerals may be may be numbered 7xx.

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENT

A portfolio analysis and management system is described. In the following description, numerous specific details are set forth such as specific formulas, etc., in order to provide a thorough understanding of the present invention. It will be obvious, however, to one skilled in the art that the present invention may be practiced without these specific details. In other instances, well-known circuits, structures and techniques have not been shown in detail in order not to unnecessarily obscure the present invention.

TERMINOLOGY

Prior to describing the present invention in detail, it is worthwhile to discuss and define certain terminology used in the following description. This definition is useful not only for purposes of an understanding of the particular processes of the present invention; but also for an appreciation of the scope of the invention. For example, the term "financial instrument" is used throughout this description and in the claims; although used in its singular form and unless stated otherwise, this term may be taken to mean a single financial instrument (e.g., a bond); a plurality of financial instruments having substantially identical maturities and coupons; or a plurality of diverse financial instruments, often termed a portfolio.

Financial Instrument: Refer to one or more of the many devices representing financial interests such as bonds and similar securities, and options on these devices. The present invention is particularly applicable to any of a number of fixed income devices and/or options on such devices. Particular ones of such devices are discussed in greater detail below. When used in the following description unless apparent to the contrary, the singular term "financial instrument" shall be taken to mean either a single financial instrument or a portfolio of such instruments including a portfolio comprising individual financial instruments of various types, maturities and coupon values.

Portfolio Manager: Refers to the individual or group of persons responsible for analyzing, valuing, and making decisions regarding the acquisition or disposal of a financial instrument regardless of whether such a portfolio manager is responsible for managing a large portfolio such as an investment fund, pension fund, etc., or the portfolio manager is an individual responsible for management of personal investments.

Duration: Refers to a tool for comparing interest rate risk between financial instruments with differing cash flows (e.g., coupons and maturities) and summarizing in a single number the sensitivity of the financial instrument to interest rate risk. A classical definition of duration is the weighted average of term to maturity of the cash flows of a financial instrument where the cash flows are expressed in terms of their present values. Duration is also known as Macaulay duration to distinguish from modified duration as defined below.

Modified Duration: Refers to a measure of the expected price change associated with financial instrument associated with a one basis point (BP) change in interest rates. Modified duration is also referred to as basis point values (BPV). A formula for calculation of modified duration is given as:

$$\text{Duration} / (1 + \text{yield/frequency})$$

Duration will be discussed in greater detail below.

Convexity: Convexity may be thought of as a second-order measure of the sensitivity of a financial instrument to changes in interest rates. Convexity will be discussed in greater detail below.

Yield: Refers to the percentage return on an investment.

Volatility: Refers to the degree to which interest rates fluctuate from period-to-period (e.g., day-to-day, week-to-week); and specifically to the variance of such flucuation. As will be seen, the present invention utilizes volatility as a stochastic factor in assessment of the exposure of financial instruments to interest rate changes. In the preferred embodiment, volatility is specifically defined as the variance (and more specifically, as the square root of the variance, e.g. its standard deviation) of the changes in the interest rate of a given maturity, expressed on an annualized basis. It might be noted that in a number of references (e.g., Fabozzi et al.), the term volatility is used; however, in these references, the term volatility is used either: (a) to refer to volatility of the price of financial instruments themselves, or (b) to discuss a principle reason for changes in pricing of financial instruments, e.g., interest rates change over time (in other words, interest rates are volatile); importantly, these references discuss assessing the exposure of financial instruments to interest rate changes in terms of duration and convexity and such references have not considered volatility of interest rates as a separable, stochastic factor in this assessment.

Volatility Exposure Value: Refers to a calculated value, based on interest rate volatity, representative of the percentage change in valuation of a financial instrument

resulting from a one unit change in volatility. The term volatility exposure value is also referred to herein as the implied market volatility.

OVERVIEW

It has long been accepted that the movement of interest rates represents the single greatest source of returns and risks in fixed income and similar financial instruments. Through various techniques portfolio managers have attempted to capitalize on the fluctuation of interest rates and the influence of such fluctuation on the returns and risks of a financial instrument. One well-known technique for measurement of sensitivity of a financial instrument to interest rate fluctuations is duration. It is well-established that duration explains a significant portion of the percentage change in the value of a financial instrument as interest rates change. However, it is also well established that duration, in and of itself, does not explain the entire price movement of a financial instrument due to interest rate changes. Therefore, a second-order measure of price sensitivity of a financial instrument to interest rate changes is also well known—convexity.

Using these two measures of price sensitivity, it has been said that "the percentage change in price [of a interest rate sensitive financial instrument] equals the sum of two components, (1) the percentage change in price due to convexity, and (2) the percentage change in price due to duration." (Concepts and Applications at page 8). Duration and convexity will be described in greater detail below. However, duration and convexity do not completely describe interest rate sensitive price changes in financial instruments; as will be seen, it has been discovered by the present invention that there an additional risk measure, volatility of interest rates (i.e., the degree to which interest rates fluctuate over time), which should be considered and which perhaps completes the equation for determining the price sensitivity of a interest rate sensitive financial instrument to changes in interest rates.

REVIEW OF CALCULATION OF THE DURATION OF AN FINANCIAL INSTRUMENT

It is useful to turn to Figures 2(a) and 2(b) to review calculation of the duration of a financial instrument. Figure 2(a) illustrates the cash flows and present values of a seven year, seven percent (7%) bond and illustrates the duration of the instrument as being approximately 5.6. Each column of Figure 2(a) represents a coupon payment 201 with the shaded portion of each column, such as shaded portion 202, representing the present value portion of the payment for the period. In addition, in the final (seventh) year, the principle repayment 203 with the shaded portion 204 representing the present value of the principle repayment. The duration value (again calculated as the weighted average term to maturity of the cash flows of the financial instrument where the cash flows are expressed in terms of their present values) may be thought of as a balance point on a seesaw, as represented by triangle 205. (This concept is explained in greater detail with reference to Concepts and Applications, at pages 2-3.)

Turning now to Figure 2(b), an illustration is provided of the duration of a seven year, fourteen percent (14%) bond. As can be seen, the duration of this instrument is slightly less than 5 years; lower than the duration of the instrument illustrated with reference to Figure 2(b) although the maturities and coupon schedule of the two instruments is identical, the difference in this exemplary case being caused by the difference in yield. The increased yield of this instrument results in a relative movement of the weighted average term to maturities of cash flows being moved forward—this weighted average being the duration.

Now, it may be worthwhile to reflect momentarily as to why duration is a significant measure. Assume the present cash value of both the instrument of Figure 2(a) and the instrument of Figure 2(b) is identical—say, \$1000.00. (The present cash values may be equal, for example, because the principle amount of the instrument of Figure 2(b) is lower than that of Figure 2(a) although the interest rate is higher for the instrument of Figure 2(b)). However, as is indicated by the duration calculation, the instrument of Figure 2(b) will, as a weighted average, produce cash in

hand at an earlier date. Therefore, the cash will be available for reinvestment in other financial instruments. Assume that a portfolio manager wishing to choose between purchasing the instrument of Figure 2(a) or the instrument of Figure 2(b) believes interest rates will generally increase between today and the maturity of the two instruments in seven years. In this case, and again assuming equal present values, the portfolio manager will choose to purchase the instrument of Figure 2(a) so as to have cash at a relatively earlier date to reinvest at the anticipated higher interest rates. If instead the portfolio manager expects a general decline in interest rates, the manager will choose the instrument with the higher duration.

Therefore it is seen how duration may be used to compare financial instruments having differing coupon rates and maturities. However, the duration calculation does not provide significant guidance as to the relative risks and potential returns of the two instruments; in other words, what is the percentage effect on today's price of the instruments of a one-percent change in interest rates? Modified duration partially provides an assessment of the relative risks and potential returns of an instrument by expressing a value representative of the percentage interest rate dependent price change of a financial instrument due to a one point change in the interest rate.

To repeat the formula for calculation of modified duration:

Duration / (1 + yield/frequency) * Portfolio market value * 0.0001

Therefore, the financial instrument of Figure 2(a) has a modified duration of:

5.6 / (1 + 0.07/1) = 5.23

reflecting an expected change in the market price of the instrument of 5.23 percent for each percentage point change in interest rates. The financial instrument of Figure 2(b) has a modified duration of:

5.0 / (1 + 0.14/1) = 4.38

reflecting an expected change in the market price of the instrument of 4.38 percent for each percentage point change in interest rates. Thus, based on modified duration calculation, the

financial instrument of Figure 2(a) offers larger potential rewards in the event of decreases in interest rates with corresponding larger potential risks in the event of increases in interest rates.

REVIEW OF CALCULATION OF THE CONVEXITY OF AN ASSET

As can be appreciated, the above-described duration calculation indicates a constant relationship between interest rates and financial instrument price. Such a constant or linear relationship is represented by the straight line 301 of Figure 3. However, relatively larger changes in yield have been shown to generally have a relatively larger effect on price and a typical price/yield curve is actually more convex than straight line, as illustrated by convex curve 302. In order to more accurately predict price, this convex nature of the price/yield curve may be taken into account—this risk measure is often termed convexity. As can be seen, for relatively small yield changes, the duration may be used to closely approximate the convex curve. However, for larger changes, such as a change from yield Y 303 to new yield 304; the straight line approach of duration introduces significant error in price prediction. This error, represented by line 305, is due to the convex nature of the relationship between price and yield.

CALCULATION OF THE VOLATILITY EXPOSURE OF AN ASSET

Two components have now been discussed which are useful for determining the relationship between interest rates and the price of financial instruments. However, the present invention has recognized a third component which influences this relationship—the volatility of interest rates. Although it has long been recognized that pricing of financial instruments is influenced by movement of interest rates, heretofore, methods for assessing the exposure of financial instrument to interest rate have merely taken into account the amount of change in interest rates as a risk measure and not the relative volatility of such changes.

The following example shows why interest-rate volatility is critically important in determining the value of bonds. Consider a bond with a two-year call option. The value of this

option on the bond depends on the volatility of the two-year rate. Volatility measures the band in which the two-year rate will probably change. Clearly, as volatility increases, this band becomes wider, and the option becomes more valuable. Stated another way, if interest rates have an higher chance of decreasing, the option is worth more to its issuer. Measuring the volatility of interest rates is thus crucial to valuing the instrument as a whole.

Measuring volatility is thus critically important to fixed-income portfolio managers. Interest-rate volatility changes over time, just as interest rates themselves change. During some periods, interest rates are relatively stable and exhibit low volatility. In other periods, the market moves up and down and exhibits higher volatility. Just as the portfolio manager needs to know the effect that changing interest rates have on portfolio value (as may be at least partially determined through use of duration and convexity measurements), it is important to be aware of the effect of changes in volatility the value as well.

It has been discovered that volatility of interest rates is particularly influencial for options or securities with option-like characteristics such as callable bonds; however, the volatility of interest rates affects the pricing behavior of all fixed-income financial instruments to some extent. In fact, the methods of present invention may be applied to assess the exposure of a large number of financial instruments, and a large number of financial decisions, to interest rate changes. A number of these financial devices and decisions are detailed in the section subtitled Applications of the Present Invention, below.

The volatility management process may be viewed as a five-step process which is illustrated with reference to Figure 4.

Term Structure

Initially, in order to consider volatility in the pricing of financial instruments, it is necessary to develop a model of the term structure of interest rates in which volatility is a stochastic factor, block 401. In the preferred embodiment, the model of the term structure recognizes one of the driving influences in fixed income financial instrument pricing is the level of interest rates; the

model of the preferred embodiment calls for a theory in which volatility constitutes a stochastic factor in addition to the interest rate factor. The development of the theory preferably involves specification of a joint stochastic process for the two factors (level of interest rates and volatility of interest rates), Figure 5, block 501; deriving the equilibrium condition, block 502; specifying the nature of the risk premium, block 503; and solving the resulting differential equation for the bond price, block 504. As one aspect of the present invention, such a model of the term structure has been developed and is given below in the section subtitled Volatility as a Stochastic Factor in Term Structure Models.

Volatility Exposure

After having determined a model of a term structure having volatility as a stochastic factor, it is then necessary to determine volatility exposure where volatility exposure is defined as the percentage change in the bond price resulting from a unit change in the volatility, block 402. As can be seen below, volatility exposure may be calculated as the partial derivative of the logarithmic price with respect to volatility and is documented as the function F below.

As one alternative to the preferred embodiment, volatility exposure may be calculated as the partial derivative of the logarithm of price (as above), however, where volatility is a deterministic parameter, as opposed to a stochastic factor. In such an alternative, it is not necessary to determine a model of the term structure of interest rates where volatility is a stochastic factor, but rather, the term structure model of the preferred embodiment may be replaced with any of a number of published term structure models having level of interest rate factors only. It might be noted that a theoretical justification for this alternative approach is lacking; however, the alternative may be expected to yield results with some practical value and offers a level of simplification over the approach of the preferred embodiment.

Computational Procedures

After having determined a volatility exposure measure, block 402, it is desirable to develop computational procedures to allow for practical application of the volatility exposure measure to

financial instruments. As will be discussed in greater detail below, the present invention has, in its preferred embodiment, implemented a computer-based application for computation of the volatility and duration of a financial instrument, and in specific, for computation of the volatility and duration of a portfolio of fixed income investments.

Prior to discussing the implementation of the preferred embodiment in detail, it may be worthwhile to provide an overview of the necessary computational procedures. First, after having determined a volatility exposure measure, it is desirable to develop mathematical procedures for computing functions for the duration exposure (D) and the volatility exposure (F), block 601. In the preferred embodiment, these procedures are defined as discussed below in connection with the subsection titled Volatility as a Stochastic Factor in Term Structure Models.

Next, it is necessary to develop apparatus and methods for calculation of the financial instrument mean and variances of return resulting from a given set of factors and the corresponding exposure measures, block 602. Such apparatus and methods are preferably implemented in the computer system described below.

Finally, it is necessary to estimate the various parameters for entry into the stochastic equations for duration and volatility and the specification of risk premium. These estimates then need to be applied to the appropriate data series to produce numerical values.

Forming Expectations

As prices of financial instruments are determined by the level of interest rates and the volatility rate, a portfolio manager wishing to assess the expected returns and risks from holding a financial instruments must form a coherent view of the development, over the next holding period, of interest rates and their volatilities, block 404. As interest rates and volatility are not forecast with certainty, the portfolio manager will typically forecast in the form of a probablistic description, i.e., with a likelihood of x, interest rates will increase by z, and volatility will increase by w. With respect to interest rate levels, the portfolio manager may choose to accept market implicit forecasts (i.e., forward rates adjusted for risk premium), or he may form an opinion

separate from such forecasts. As to the variance of interest rates (the number represented by volatility), the portfolio manager first determines what level of volatility is expected by the market.

The application of levels of expectation to volatility exposure is illustrated with reference to Figure 7. Initially, a portfolio manager may form an expectation E1 as to the level of volatility in interest rates during a relevant holding period, block 701. The portfolio manager may then form an expectation E2 as to the expected level of interest rates during the holding period, block 702. The expectation E1 and expectation E2 are applied to a formula F (given below, in the subsection titled Volatility as a Stochastic Factor in Term Structure Models), blocks 703 and 704. The result of formula F is an output V representative of the volatility exposure level of the instrument during the relevant period, block 705, where the volatility exposure value V is representative of the percentage change in valuation of the financial instrument resulting from a one unit change in the volatility of interest rates. The portfolio manager may then evaluate the relative risks and potential rewards of the financial instrument based on the volatility exposure level V, block 706.

Implementation in an investment strategy

Finally, the portfolio manager must implement an investment strategy. Implementing an investment strategy in general comprises the step of constructing and maintaining a set of financial instruments (or a portfolio) that represents as nearly optimal position as possible with respect to the returns and risks which are attributable to expected changes in the level of interest rates and the volatility of interest rates.

The specifics of implementing a strategy vary depending on the particular investment objectives. For example, for general accounts, the objective may be to invest in financial instruments which represent an efficient tradeoff between risk and return, i.e, the portfolio can not be improved upon in terms of expected return without taking additional risk and, the level of risk can not be improved (lowered) without reducing expected return. Levels of risks and returns in such a case are evaluated taking into account both interest rate and volatility factors.

A different situation arises when the portfolio manager desires a return which compares favorably with a specific benchmark, e.g., the performance of an index. In such a situation, the objective is to minimize the variance of the difference between the benchmark and the portfolio return.

In other cases, the benchmark may be set by the fund liabilities, such as certificates of deposit or guaranteed income securities. In these cases, the objective may be to maximize the probability of funding the liabilities.

A particularly pervasive situation is that of pension plan funding where the assets are supposed to be sufficient to fund payments to beneficiaries when due. A popular technique, termed immunization, eliminates or minimizes interest rate risk; however, prior art methods do not provide any protection against volatility risk. The present invention provides tools for immunizing portfolios against both sources of risk. This closer control may have the effect of extending investment opportunities by allowing the portfolio manager to consider investments that would otherwise be considered incompatible with the objects (i.e., callable bonds).

Certain other applications of the present invention are detailed immediately below. These applications include hedging, where the hedges are constructed with regard to both interest rate and volatility factors; construction of synthetic securities (that is, portfolios which mimic the behavior of a given security under all circumstances but which may allow for an increased return under certain of the circumstances); and performance measurement where, by applying the methods of the present invention, realized results can be attributed to the proper sources.

APPLICATIONS OF THE PRESENT INVENTION

Although much of the above discussion has related to the application of the methods of the present invention to assessment of a fixed income financial instrument, it should be noted that the present invention has application to assessment of other assets. For example, the present invention has application in the following areas:

(A) Individual asset assessment applications:

(1) Valuation of default-free securities;

(2) Valuation of future contracts;

(3) Valuation of options including options on future contracts and fixed income investments;

(4) Contingent claims analysis;

(5) Synthetic security analysis; and

(6) Mortgage backed security valuation;

(B) Portfolio assessment applications:

(1) Portfolio optimization;

(2) Portfolio hedging;

(3) Portfolio indexing;

(4) Immunization;

(5) Active Management;

(6) Liability Funding; and

(7) Asset/liability management.

IMPLEMENTATION OF THE CURRENTLY PREFERRED EMBODIMENT

The computer system of the preferred embodiment

The present invention preferably utilizes a computer system to perform certain data manipulation and storage. By way of overview, it may now be useful to provide description of the computer system of preferred embodiment and such a computer system is described with reference to Figure 1. The preferred embodiment of the present invention is implemented on one of members of the IBM Personal Computer family or computer systems compatible with members of this family. In particular, the computer system of the present invention is implemented on a IBM

PC capable of executing the OS/2 operating environment. As will be familar to one of ordinary skill in the art, this operating system includes windows, pull-down menus, buttons, and mouse support, all of which increase the user-friendliness of the system. It will, of course, be obvious to one of ordinary skill in the art that any number of other computer systems may employ the teachings of the present invention.

In any event, a computer system as may be utilized by the preferred embodiment generally comprises a bus or other communication means 101 for communicating information, a processing means 102 coupled with the bus 101 for processing information, a random access memory (RAM) or other dynamic storage device 104 (commonly referred to as a main memory) coupled with said bus 101 for storing information and instructions for said processor 102, a read only memory (ROM) or other static storage device 106 coupled with said bus 101 for storing static information and instructions for said processor 102, a display device 122 , such as a cathode ray tube, liquid crystal display, etc, coupled to the bus 101 for displaying information to the computer user, an alphanumeric input device 125 including alphanumeric and other keys coupled to said bus 101 for communicating information and command selections to said processor 102, and a cursor control device 127, such as a mouse, track ball, cursor control keys, etc, coupled to said bus 101 for communicating information and command selections to said processor 102 and for controlling cursor movement. Finally, it is useful if the system includes a hardcopy device 129, such as a printer, for providing permanent copies of information. The hardcopy device 129 is coupled with the processor 102, main memory 104, static memory 106 and mass storage device 107 through bus 101.

The processor 102 of the preferred embodiment is one of the 80x86 microprocessor family manufactured by Intel Corporation of Santa Clara, California, and more specifically is an 80386 processor. The current implementation of the present invention requires at least 4 megabytes of main memory 104 for proper execution. Further, significant advantages are gained through use of a relatively high density and high speed mass storage device such as a hard (e.g., fixed) disk over

use of relatively less dense and slower devices, such as typical removable (e.g., floppy) disks. The current implementation of the present invention requires approximately 1 megabyte of disk space for installation.

It will be apparent, from an understanding of the present invention from the below description, that several of the above-mentioned components of the computer system of the preferred embodiment are not essential to operation of a computer system employing aspects of the present invention. For example, the cursor control device 127, and hard copy device 129 may not be present in certain implementations.

As will be understood, the present invention may utilize the processor 102 for processing instruction; for example, as a means for processing a volatility exposure value. The present invention may further utilize the mass storage device 107 to store certain information; for example, as a means for storing certain data inputs, such as default data inputs. Further, the present invention utilizes the keyboard 125 and cursor control device 127 as data input means allowing input of data to be processed. Still further, the system of the present invention utilizes main memory 104 as a means for storing data; for example, for storing a calculated volatility exposure value.

The current implementation of the system of the present invention

The current implementation of the present invention allows portfolio managers to quantify the effect of changes in volatility on portfolio value. The system provides the portfolio manager with an implicit market forecast of interest rates as well as volatility, and allows the portfolio manager to override these inputs with forecasts of their own. The system also recognizes spreads as an additional source of risk in the fixed income markets.

The current implementation uses volatility forecasts or implied market volatility (along with other inputs discussed in greater detail below) to determine the optimal portfolio allocation among various asset classes. The optimal allocation minimizes the interest-rate risk, volatility risk, and spread risk at given levels of expected return subject to desired portfolio characteristics. In contrast

to traditional models, the model used in the system does not make the assumption that the yield curve must shift in a parallel fashion to maintain the integrity of model results. Partial durations and convexities, which measure exposure to interest-rate movements at a given maturity, allow the model to depict more realistic movements in interest rates.

The end result is an improved assessment of portfolio risk that can be used by both passive and active portfolio managers.

Application to Portfolio Management

The current implemention of the present invention can be used with any of the following five investment strategies:

1. Portfolio Optimization: Allows portfolio managers to maximize return at a given level of risk, subject to portfolio constraints;
2. Portfolio Indexing: Allows portfolio managers to construct a portfolio that optimally tracks, and perhaps outperforms, a given index;
3. Liability funding: Allows portfolio managers to optimally construct an immunized portfolio that can be used to fund a given liability stream;
4. Asset/Liability Management: Allows portfolio managers to construct positions that match the characteristics of assets and liabilities; and
5. Portfolio Hedging: Allows the portfolio manager to construct synthetic securities or lock in a hedge.

Each portfolio mentioned above consists of general securities that represent asset classes. Futures and options can also be processed as an asset class if the portfolio manager's strategy uses these instruments in the management process.

Of course, it will be obvious to one of ordinary skill in the art that alternative embodiments of the present invention may address others of the areas of application of the present invention listed in the section subtitled Applications of the Present Invention, above.

Data files

In the current implementation of the present invention, certain data files are provided to the user either at the time of installation of the system or on a supplemental basis after installation and are stored on the mass storage device 107 for use during practice of the methods of the present invention. In particular, two of these data files are:

CORCOM.DAT This data file contains a correlation matrix, market price of risk and relative volatilities for all factor maturity points. The data in this file is based on historical data and is utilized in calcuation of the volatility exposure; in the current implementation, the assignee of the present invention may provide updates to this file on a non-periodic basis. Of course, such data may be independently calculated and utilized in addition to obtaining the information through the assignee of the present invention.

YYMMDD.RTS These files contain weekly and last business day of the month's forward rates files. These rates are calculated from Friday's and/or the last business day's closing prices. These files may be, for example, provided by the assignee of the present invention each Monday or the first day of each month. In the current implemenation, these files are provided via mail on a monthly basis but can also be accessed electronically on a weekly basis through computer links with computer systems maintained by the assignee of the present invention. Of course, this data may be independently gathered and utilized in addition to obtaining the information through the assignee of the present invention.

Operation of the current implementation of the present invention

Perhaps the most expediant way of describing in some detail the current implementation of the present invention is by way of an example illustrating the steps involved in a sample run. Such

a sample run may be explained with reference to Figure 8. Initially, the user of the system inputs certain information such as specifications, objectives, and parameters, block 801. The input of specifications, objectives and parameters is discussed in greater detail with reference to Figure 9 which shows, initially, a title for the run is specified by the user, block 901. In the current implementation, the title for the run is specified by positioning a cursor on the screen, using the cursor control device 127, in an area on the display 122 labeled TITLE and typing, using the keyboard 125, the desired title. The particular title may be any user chosen name for the run fitting certain limitation (e.g., alphanumeric characters, length of the title, etc.) which very well may vary from implementation to implementation.

As will be understood, other data input steps which will be discussed in greater detail below may be carried out utilizing the cursor control device 127, display 122 and keyboard 125 in conventional manners and, further, other data inputs are subjected to certain limitations and data editing steps which are well known in the art such as number of characters in the field, requirements for a given field to be numeric or alphabetic, etc. In order to avoid unnecessary complexity in this description, such limitations and data editing steps will not be further described.

Next, a holding period is specified, block 902. The holding period reflects the period of time the an asset combination is expected to be held without rebalancing the portfolio. The holding period is expressed, in the current implementation, in months and may be, for example, 12 months. Of course, a different holding period may be specified based on the requirements of the particular run. In the current implementation, the holding period must be entered as a number between 0 and 12.

Optionally, a total asset value and total liability value may be input, block 903. When performing portfolio optimization or hedging, the total asset value must be entered as the total value of all asset classes in your portfolio. For portfolio indexing, liability funding, and asset/liability

management investment objectives, the system needs to know both total asset value and total liability value. In the event a total asset value is not entered, the system assumes total asset value is equal to the total liability value. If total liability value is not entered, the sytem will calculate this value by summing the current holdings (i.e., the present value) of each liability class when you run the program.

Finally, an objective of the run is specified, block 904. Currently, the system supports the objectives of Portfolio Optimization, Portfolio Indexing, Liability funding, Asset/Liability Management, and Portfolio Hedging, each of which was discussed in greater detail above.

Referring again back to Figure 8, the implied market volatility is computed, block 802. The process for computing the implied market volatility requires information on at least one bond futures contract and at least one option on that contract to compute the implied volatility. This is described in greater detail by Figure 10. Initially, the current risk-free interest rate is specified, block 1001. Next, the price of a futures contract is specified, block 1002. The price is entered in to the nearest 1/32 of a dollar. Current futures prices may be found in any of a number of publications, including the Wall Street Journal.

Next, the duration of the selected futures contract is input, block 1004. The duration of futures contracts may be calculated as described above—further, a number of automated tools are available to perform duration calcuations including a futures system available from the assignee of the present invention.

Information is then specified on an option for the futures contract, block 1005. First, the expiration date of the option is specified; then, the strike price of the option is specified; and finally, the option premium is specified. Again, the expiration data, strike price and option premiums for options may be found in a number of publications including the Wall Street Journal. In order to increase the accuracy of the implied volatility calculation, additional options may be entered including their strike prices and options premiums. In the current implementation, information on up to six options may be entered; however, all options must have the same

expiration date and must be options on the same futures contract. In the event multiple options are entered, the options may be weighted based on their relative weights in indicating implied market volatility.

The implied market volatility is then calculated utilizing the methods described below in the section subtitled Volatility as a Stochastic Factor in Term Structure Models, block 1006, and the resultant implied market volatility value is displayed on the display 122.

It is noted that in addition to calculating the implied market volatility, the current implementation of the present invention allows calculating option premiums given a certain assumed level of implied volatility.

To calculate option premiums, the following information should be input: general information including (a) risk-free rate, (b) evaluation date, and (c) implied market volatility; information on the underlying security including (a) security type (it may be a bond or a futures contract), and (b) price of the security; and information on the option including (a) the expiration date and (b) the strike price.

Next, correlation matrix defaults may be modified, block 803. As was mentioned above, in the preferred embodiment, a correlation matrix file is provided which comprises correlation coefficients obtained through studies of historical data. This data may be used as supplied in the CORCOM.DAT file or may be modified for an individual run. These coefficients measure the degree to which movements in interest rates at a given maturity date relate to the measure of movements in interest rates at another maturity date.

In addition to modifying the correlation matrix defaults, the current implementation allows for modifying other parameters including (1) relative volatilities, which measure the volatility of interest rates at maturity dates relative to the volatility of the long rate; (2) long rate volatility; and (3) market price risk, which is a measure of required compensation, in units, that an investor must receive for each unit change in the level of risk.

The expected interest rates may be modified for each maturity point, block 804. The default expected interest rates are taken from the YYMMDD.RTS file and displayed on the display 122.

Next, the current implementation allows input of information on specific assets and liabilities to be evaluated, block 805. The system allows input of three types of assets: (1) regular assets; (2) futures; and (3) options. Entering assets is described in greater detail with reference to Figure 11 which illustrates the steps for entering a regular asset. Initially, an asset name is entered, block 1101. In the current implementation two formats of the asset name may be entered—a long format and a short format—allow for increased ease of reference to the asset class during operation of the system of the current implementation. Next, an upper limit (maximum) and lower limit (minimum) amount of the asset class to be held in the portolio is specified, block 1102 and 1104. These numbers are used during calculations to limit increases in holdings of this asset class when reaching an optimal solution. Next, the average maturity data for assets in the class is specified, block 1105; then, the number of years until the first call date for assets in the class, block 1106; the weighted average life of assets in the class, block 1108; the weighted average life of assets in the class, block 1109; the average duration and average convexity, block 1112; the average coupon rate, block 1113; and average spread, block 1114, are specified.

Additional regular assets may be entered in a similar fashion, specifying a unique name of the asset class, block 1101, for each asset class entered. Asset classes may be updated by specifying the name of the asset class to be updated and updating the desired parameters. Asset classes may also be deleted by specifying the name of the asset class to be delete and indicating a delete function.

Futures may be input, updated and deleted in a similar fashion also; however, only the name, block 1101, upper limit, block 1102, and lower limit, block 1104, are entered for futures. The price, duration and convexity information is taken from the futures contract information entered earlier (see Figure 10).

Options are entered similar to regular assets; however, again only the name, block 1101, upper limit, block 1102, and lower limit, block 1104 are entered.

Liabilities are entered in one of two forms; in the form of averages or in the form of cashflows. Liabilities entered in the form of averages is illustrated in greater detail with reference to Figure 12(a); liabilities entered in the form of cashflows in illustrated in greater detail with reference to Figure 12(b). Referring first to Figure 12(a), blocks 1201-1204 correspond to blocks 1101-1104 of Figure 11 wherein a name of the liability is entered, block 1201; the upper limit for holdings of the liability is entered, block 1202; and the lower limit for holdings of the liability is entered, block 1204. Next, the present value and current holdings of the liability stream is entered, block 1205 and the stated maturity is entered, block 1206.

Alternatively, and with reference to Figure 12(b), liabilities may be entered in terms of their cashflows. Initially, a liability name is entered, block 1221. Then for each cash outflow, the year of the cash outflow is entered, block 1222, followed by the dollar amount of the cash flow, block 1224. This process continues for each cashflow of the liability class and then additional liability classes may be entered by entering the class name, followed by the year and dollar amounts of cash flows.

Finally, the current implementation allow specifying of certain optimization settings, block 806, including the number of efficient portfolios to be calculated and the target return. The system will calculate the probability that the expected portfolio returns will be less than the target returns.

The system is then run against the input data and the outputs reviewed, block 807. Five output windows may be displayed on display 122: (1) the optimal portolio window; (2) the asset / liability expectations window; (3) the asset / liability characteristics window; (4) the portfolio composition window; and (5) the portfolio characteristics window. The results displayed in any of these windows may be printed on hard copy device 129.

Each of the windows displays the title of the run, the holding period, the objective, and the evaluation date, as input and described above. The optimal portfolio window displays the holding

of each asset class in the calculated optimal portfolio where the optimal portfolio is defined as the efficient portfolio that has the minimum amount of risk of all of the efficient portfolios calculated by the system. The following data appears on this window:

Expected Return:	The expected return of the asset portfolio or the expected mismatch between the return of the asset portfolio and the return of the liability portfolio.
Standard Deviation:	The standard deviation of the expected return described above.
Asset Duration Total:	The total duration of all asset classes combined.
Asset Duration at Factor Maturities:	The partial asset durations at each factor maturity point.
Liability Duration Total:	The total duration of all liability classes combined.
Liability Duration at Factor Maturities:	The partial liability durations at each factor maturity point.
Asset Convexity Total:	The total convexity of all asset classes combined.
Asset Convexity at Factor Maturities:	The partial asset convexities at each factor maturity point.
Liability Convexity Total:	The total convexity of all liability classes combined.
Liability Convexity at Factor Maturities:	The partial liability convexities at each factor maturity point.
Asset Volatility Exposure:	The asset portfolio's exposure to changes in market volatility. Liability Volatility Exposure The liability portfolio's exposure to changes in market volatility.
Asset/Liability:	The name of the asset or liability class.
Duration:	The duration of the asset or liability class.
Convexity:	The convexity of the asset or liability class. Duration and convexity are input by the user for all asset classes and liability classes input as

	averages. The system calculates this number for liability classes input as cashflows.
Volatility Exposure:	The volatility exposure for each asset and liability class.
Current Holdings:	The current holdings optionally input by the user for each asset and liability class.
Optimal Portfolio:	The holdings of each asset class in the optimal portfolio. The optimal portfolio displayed on this screen is the efficient portfolio that has the minimum amount of risk among all of the efficient portfolios calculated by the system.
Return:	Target returns entered as optimization parameters before running the analysis. As part of the analysis process, the system calculates probability that returns will be lower than the specified target returns. Return is defined as the expected return of all asset classes if only asset classes were defined in the input for the analysis. If a liability class was defined, return is the mismatch between asset returns and liability returns. If this return is positive, asset cashflows will outweigh liability cashflows. If return is negative, liabilities will exceed assets.
Probability:	The probability that returns will be lower than the target the return.

The asset / liability expectations window displays data on the expected return of each asset class over the expected holding period. The following data is displayed on this window:

Expected Return:	The expected return of each asset class over the specified holding period.
Standard Deviation:	The standard deviation of expected return for each asset class.
Correlation Matrix:	The correlation of expected return between asset classes. Correlation is the degree to which the return of two classes are related.

The asset/liability characteristics window displays certain characteristics of the assets and liabilities. This window displays the following data:

Total Duration:	The total duration of the asset or liability class.
Total Convexity:	The total convexity of the asset or liability class.
Total Volatility Exposure:	The volatility exposure for each asset and liability class.
Duration Components:	The partial durations for each asset and liability class at each factor maturity point.
Convexity Components:	The partial convexities for each asset and liability class at each factor maturity point.

The portfolio composition window displays data on calculated efficient portfolios. The following information is displayed on this window:

Portfolio Number:	The system calculates a number of efficient portfolios based on volatility, term structure, and asset and liability information. The efficient portfolios listed on this screen are listed in the order of increasing risk; each asset and liability combination represents an efficient portfolio. Each efficient portfolio is identified by a portfolio number The portfolio numbered one will always have the lowest risk (i.e., lowest standard deviation). The system may not calculate the exact number of efficient portfolios input before running the analysis. The algorithm that the system uses may create a few more or a few less portfolios, depending upon the optimization process.
Expected Return:	The expected return of each portfolio on the efficient frontier.
Standard Deviation:	The standard deviation of the expected return above.
Asset/Liability Class Names:	The holdings of each asset class in each of the efficient portfolios.
Target Return:	The five target returns input before running the analysis.
Probability:	The probability that returns of each efficient portfolio will be lower than the target return that you specified before running the analysis.

Finally, the portfolio characteristics window provides data on characteristics of the calculated efficient portfolios. The following data is displayed on the window:

Portfolio:	The number of the efficient portfolios. Each portfolio number corresponds to the portfolio with the same number on the portfolio composition window.
Total Duration:	The total duration of each efficient portfolio.
Total Convexity:	The total convexity of each efficient portfolio.
Total Volatility Exposure:	The volatility exposure for each efficient portfolio.
Duration Components:	The partial durations for each efficient portfolio.
Convexity Component:	The partial convexities for each efficient portfolio.

VOLATILITY AS A STOCHASTIC FACTOR IN TERM STRUCTURE MODELS

It has been noted by fixed-income practitioners that the volatility of interest rates changes in time. There are periods when the bond market is very volatile, with interest rates changing drastically from day to day or from hour to hour. At other times, the fixed-income markets are fairly quiet, with only gradual changes in interest rates. Typically, rates are more volatile when their level is high, and less so in periods of generally low interest rates. The level of rates, however, does not alone explain the differences in their volatility.

Table 1 contains data on interest rate volatility over the period 1987-1989. The column labeled "Actual Variance" contains an estimate of the variance of changes in the yield on long U.S. Treasury bonds within each month. The numbers were obtained as the annualized variance over the month of daily returns on the nearest Treasury bond futures contract, divided by the duration of the futures contract. Even though the numbers are estimates (subject to an estimation error of about 25% of their values), it is apparent that the volatility changes from month to month. These changes are recognized, and to an extent anticipated, by the market. The column "Implied Variance" provides the variance of the long Treasury rate calculated from the premia on listed calls on the Treasury bond futures contract as of the last day of the previous month.

Table 1.
Actual and Implied Volatility of Long Treasury Rates

Month	Actual Variance x10000	Implied Variance x10000
1/87	0.94	1.39
2/87	0.84	1.22
3/87	0.98	0.84
4/87	3.51	1.02
5/87	4.17	2.11
6/87	2.85	1.81
7/87	1.30	1.57
8/87	2.14	1.52
9/87	2.57	1.38
10/87	8.94	2.00
11/87	1.44	3.57
12/87	3.38	2.72
1/88	4.15	2.44
2/88	1.16	1.98
3/88	1.30	1.36
4/88	1.00	1.75
5/88	0.65	1.66
6/88	3.34	1.30
7/88	1.02	1.68
8/88	1.11	1.39
9/88	1.72	1.39
10/88	1.10	1.35
11/88	1.00	1.38
12/88	1.50	1.28
1/89	0.66	1.16
2/89	0.98	1.10
3/89	1.08	1.02
4/89	0.92	0.90
5/89	1.52	0.81
6/89	1.69	1.01
7/89	0.56	1.01
8/89	1.10	0.90
9/89	0.51	0.97
10/89	0.58	0.90
11/89	0.49	0.84
12/89	0.26	0.68

It has been likewise realized that interest rate volatility affects prices and returns of fixed-income securities. This is most apparent on interest rate options and instruments with embedded option features, such as callable bonds or mortgage-backed securities. Indeed, all results dealing with pricing of options, starting with the Black/Scholes formula, show strong dependence of the option price on the variance of the underlying process. Even non-callable

bonds, however, are volatility dependent. The available theories of the term structure of interest rates (cf., for instance, Vasicek (1977), Cox, Ingersoll, and Ross (1985), Heath, Jarrow, and Morton (1988), Brennan and Schwartz (1979), etc.) all show the presence of a variance parameter in the bond pricing formula.

The practical impact of these contentions is that if the volatility increases unexpectedly, the price of a fixed-income instrument with a positive volatility exposure will instantly increase for a capital gain. An unexpected decrease in the interest rate volatility will generate a capital loss. If the volatility exposure of an investor's portfolio differs from that of a bond index or other benchmark of performance, the capital gains or losses will not match those of the benchmark and may result in a substandard performance. Like changes in interest rate level, a change in volatility of rates is a source of risk and returns.

We wish to develop a framework in which interest rate volatility is explicitly recognized as a stochastic factor, and to determine the pricing of fixed-income securities and the term structure of interest rates consistent with a market equilibrium within that framework.

This is indeed necessary in order to determine the role of changing volatility in fixed-income markets. We cannot postulate a term structure model in which volatility is a deterministic parameter, and study the effect of changes in that parameter. Varying the parameter would violate the assumption of such model that the parameter is constant, and therefore invalidate its conclusions.

This paper presents a two-factor model of the term structure, in which the factors are the short rate and its instantaneous variance. The pricing of fixed-income instruments is derived under the equilibrium condition of no arbitrage. The exposure of security prices to each of the factors is determined, resulting in identification of duration and volatility exposure as the measures of risk. Implications for management of fixed-income portfolios are discussed in the context of both active and passive investment strategies.

The Model

Consider a term structure of interest rates in which the short rate r follows a diffusion process described by

$$dr = \alpha(\bar{r} - r)\,dt + \sqrt{v}\,dz$$

where x is a Wiener process with unit variance,

$$E(dx)^2 = dt .$$

The short rate shows a tendency to revert to a central value $\bar{r}$, with the reverting force $\alpha(\bar{r} - r)$ proportional to its current deviation from the mean, and an instantaneous variance v. It may be noted that there is a non-zero probability, however small, for the short interest rate to reach negative values, and therefore this specification can serve only as an approximation to an economically meaningful market.

If the variance v is constant, a one-factor description of the term structure can be derived after specifying the pricing of risk in the market. We will, however, assume that the variance is itself a stochastic process. Specifically, it will be supposed that it follows the process

$$dv = \gamma(\bar{v} - v)\,dt + \xi\sqrt{v}\,dy$$

where $\bar{v} > 0$ is a long-term average value of v, $\gamma > 0$ is the proportionality constant of the mean reverting force, and $\xi^2 v$ is the instantaneous variance, proportional to the level of v. The Wiener process y is assumed to have a unit variance and correlation ρ with the process x,

$$E(dy)^2 = dt$$

$$E(dx)(dy) = \rho\, dt .$$

The variance v of the short rate possesses a stationary distribution concentrated on strictly positive values whenever

$$\gamma\bar{v} \geq \tfrac{1}{2}\xi^2 .$$

The stationary mean and variance of v are $\bar{v}$ and $\frac{1}{2}\xi^2\bar{v}/\gamma$, respectively.

Alternatively, we can describe the volatility of the short rate r in terms of its instantaneous standard deviation $\sigma = \sqrt{v}$, which is then subject to the process

$$d\sigma = \tfrac{1}{2}\left(\left(\gamma\bar{v} - \tfrac{1}{4}\xi^2\right)\tfrac{1}{\sigma} - \gamma\sigma\right)dt + \tfrac{1}{2}\xi\, dy .$$

The standard deviation has a constant instantaneous variance, and stays positive in virtue of the drift term, which increases beyond bounds as σ approaches zero.

The current values of r=r(t) and v=v(t) fully determine the probability distribution of the subsequent development of the process r(s), v(s) for s $\geq$ t. If, moreover, the risk premia in the market are functions of r and v only, as we will assume, then the short rate and its variance constitute the two factors that specify the pricing of bonds and the structure of interest rates.

Let P(t,s) = P(t,s,r,v) be the price at time t of a bond with a unit maturity value at s $\geq$ t, given the current values r(t) = r, v(t) = v of the two factors. The behavior of the price can be described by the equation

$$\frac{dP}{P} = \mu\, dt - \varphi\, dx + \psi\, dy$$

where

$$\mu = \frac{1}{P}\Big(P_t + \alpha(\bar{r} - r)P_r + \gamma(\bar{v} - v)P_v + \tfrac{1}{2}vP_{rr} + \rho\xi v P_{rv} + \tfrac{1}{2}\xi^2 v P_{vv}\Big)$$

is the expected rate of return on the bond, and

$$\varphi = -\sqrt{v}\, P_r / P$$
$$\psi = \xi\sqrt{v}\, P_v / P$$

are the components, corresponding to the processes x,y, respectively, of the variance $\varphi^2 - 2\rho\varphi\psi + \psi^2$ of the rate of return. We have arbitrarily chosen the signs of φ, ψ to correspond to the direction of the relationship between each factor and the price. Here and throughout, subscripts denote partial derivatives.

An arbitrage argument applied to the equation for price (see, for instance, Ross, 1977) leads to the equilibrium condition

$$\mu = r + q\varphi + p\psi$$

holding for a bond of any maturity $s \geq t$, where q, p are the market prices of risk due to interest rate changes and volatility changes, respectively. The market prices of risk are independent of the bond maturity date s.

We will assume that the risk prices are proportional to the risk level

$$q = \lambda\sqrt{v}$$

$$p = \eta\sqrt{v}$$

where λ, η are constants. This specification, together with the description of the processes r, v, fully describes the assumptions of the model.

We will assume the following ranges for the values of the parameters:

$$\alpha, \gamma, \zeta, \bar{r}, \bar{v} > 0$$

$$\lambda \geq 0$$

$$-1 < \rho < 1$$

$$\gamma\bar{v} \geq \tfrac{1}{2}\zeta^2$$

$$\gamma + \zeta\eta > \frac{\zeta}{\alpha}(1-\rho) .$$

This last condition enforces the stability of the solution.

Upon substitution of the appropriate expressions into the equilibrium condition, we obtain a partial differential equation for P=P(t,s,r,v) as

$$P_t + (\alpha\bar{r} - \alpha r + \lambda v)P_r + (\gamma\bar{v} - (\gamma + \xi\eta)v)P_v$$
$$+ \tfrac{1}{2} v P_{rr} + \rho\xi v P_{rv} + \tfrac{1}{2}\xi^2 v P_{vv} - rP = 0.$$

This equation is subject to the boundary condition

$$P(s,s,r,v) = 1.$$

Since the coefficients in the equation are assumed to be temporarily homogeneous, we will replace t by the time to maturity s-t. The term P_t is then replaced by $-P_t$, and the boundary condition will become

$$P(0,r,v) = 1.$$

The Solution

The solution of the partial differential equation has the form

$$P(t,r,v) = \exp(-rD(t) + vF(t) + G(t))$$

where D(t), F(t), and G(t) are functions of the time to maturity, subject of the conditions

$$D(0) = 0$$
$$F(0) = 0$$
$$G(0) = 0 .$$

To prove that, we get upon differentiation

$$P_t = P(-rD' + vF' + G')$$

$$P_r = -DP$$

$$P_v = FP$$

$$P_{rr} = D^2 P$$

$$P_{rv} = -DFP$$

$$P_{vv} = F^2 P$$

and on substitution,

$$\begin{aligned} & rD' - vF' - G' - (\alpha\bar{r} - \alpha r + \lambda v)D \\ & + (\gamma\bar{v} - (\gamma + \xi\eta)v)F \\ & + \tfrac{1}{2}vD^2 - \rho\xi v DF + \tfrac{1}{2}\xi^2 v F^2 - r = 0. \end{aligned}$$

We note that the left-hand side expression is linear in r,v. Collecting the separate powers produces the following three equations:

$$G' + \alpha\bar{r}D - \gamma\bar{v}F = 0$$

$$D' + \alpha D - 1 = 0$$

$$\begin{aligned} & F' + \lambda D + (\gamma + \xi\eta)F \\ & \quad - \tfrac{1}{2}D^2 + \rho\xi DF - \tfrac{1}{2}\xi^2 F^2 = 0. \end{aligned}$$

We thus see that if these three ordinary differential equations can be solved for the functions D, F, G

subject to the boundary conditions, the proposed form for P is indeed the solution of the partial differential equation.

The first equation is easily integrated for G to produce

$$G(t) = -\alpha \bar{r} \int_0^t D(\tau)\, d\tau + \gamma \bar{\nu} \int_0^t F(\tau)\, d\tau .$$

The second equation has the solution

$$D(t) = \frac{1}{\alpha}\left(1 - e^{-\alpha t}\right) ,$$

same as in Vasicek (1977).

The third equation is the most troublesome. Substituting for D, we get an equation for the F in the form

$$F' - \frac{1}{2}\xi^2 F^2 + \left(\gamma + \xi\eta + \frac{\rho\xi}{\alpha} - \frac{\rho\xi}{\alpha} e^{-\alpha t}\right) F$$

$$- \left(\frac{1}{2\alpha^2} - \frac{\lambda}{\alpha} + \left(-\frac{1}{\alpha^2} + \frac{\lambda}{\alpha}\right) e^{-\alpha t} + \frac{1}{2\alpha^2} e^{-2\alpha t}\right) = 0 .$$

This is a Riccati equation, involving the square of the function. The substitution

$$H(t) = \exp\left(-\frac{1}{2}\xi^2 \int_0^t F(\tau)\, d\tau\right)$$

allows transforming the equation into a linear second order equation for H. Indeed, we have

$$F(t) = -\frac{2}{\xi^2} H'/H$$

$$F'(t) = -\frac{2}{\xi^2}(H''H - H'^2)/H^2$$

and after substitution and rearrangement,

$$H'' + \left(\gamma + \xi\eta + \frac{\rho\xi}{\alpha} - \frac{\rho\xi}{\alpha}e^{-\alpha t}\right)H'$$

$$+\frac{1}{2}\xi^2\left(\frac{1}{2\alpha^2} - \frac{\lambda}{\alpha} + \left(-\frac{1}{\alpha^2} + \frac{\lambda}{\alpha}\right)e^{-\alpha t} + \frac{1}{2\alpha^2}e^{-2\alpha t}\right)H = 0$$

We seek a non-zero solution of this equation subject to the boundary condition

$$H'(0) = 0 .$$

Although the equation is of the second order, one boundary condition is sufficient to determine a solution up to a multiplicative constant and therefore the function F(t) is uniquely given.

A further transformation

$$t = -\frac{1}{\alpha}\log x \quad , \quad 0 < x \leq 1$$

$$H(t) = x^{\beta} Q(x)$$

reduces this equation to the form

$$xQ'' + \left(2\beta - \theta + 1 + \frac{\rho\xi}{\alpha^2}x\right)Q'$$

$$+\left(\frac{\xi\rho\beta}{\alpha^2} - \frac{\xi^2}{2\alpha^4}(1-\alpha\lambda) + \frac{\xi^2}{4\alpha^4}x\right)Q = 0$$

where

$$\beta = \tfrac{1}{2}\theta - \tfrac{1}{2}\sqrt{\theta^2 - \xi^2/\alpha^4 + 2\lambda\xi^2/\alpha^3}$$

$$\theta = \frac{\gamma + \xi\eta}{\alpha} + \frac{\rho\xi}{\alpha^2} \quad .$$

This is a homogeneous linear differential equation of the second order whose coefficients are linear functions of the variable. Every such equation can be transformed into Kummer's equation

$$zW'' + (b-z)W' - aW = 0 ,$$

whose solution $W(z)$ can be expressed in terms of confluent hypergeometric functions.

Although the solution of the equation for Q(x) is real, the transformation into the equation for W(z) involves complex values. There is no particular significance to the complex argument. It is simply the price to pay for wanting to express the solution of the equation for Q(x) in terms of functions that have been previously named and investigated. In fact, the computational procedures are highly efficient in the complex domain, with the resulting value of Q(x) (and therefore F(t)) being, of course, real.

The transformation of the equation for $Q(x)$ into Kummer's equation has the form

$$x = -iz/\varkappa$$
$$Q(x) = e^{-\delta z} W(z)$$

where

$$\varkappa = \frac{\xi}{\alpha^2}\sqrt{1-\rho^2}$$

$$\delta = \frac{1}{2} - \frac{i}{2}\frac{\rho}{\sqrt{1-\rho^2}} .$$

The values of the parameters a,b in Kummer's equation are then

$$b = 2\beta - \theta + 1$$
$$a = \frac{1}{2}b - \frac{i}{2}\frac{1}{\sqrt{1-\rho^2}}\left(\frac{\xi}{\alpha^2}(1-\alpha\lambda) - \rho(\theta-1)\right).$$

The complete solution of Kummer's equation is

$$W(z) = K_1 M(a,b,z) + K_2 z^{1-b} M(1+a-b, 2-b, z)$$

where K_1 , K_2 are arbitrary constants and

$$M(a,b,z) = 1 + \sum_{n=1}^{\infty} \frac{a(a+1)\ldots(a+n)}{b(b+1)\ldots(b+n)} \cdot \frac{z^n}{n!}$$

is the confluent hypergeometric function.

On substitution, we get the solution for the function H(t) as

$$H(t) = K_1 e^{-\alpha\beta t - i x \delta e^{-\alpha t}} M(a, b, i x e^{-\alpha t})$$

$$+ K_2 e^{-\alpha(\beta - b + 1)t - i x \delta e^{-\alpha t}} M(1+a-b, 2-b, i x e^{-\alpha t}).$$

To introduce a more compact notation, let $\beta_1 = \beta$, $\beta_2 = \theta - \beta$ be the two roots of the equation

$$\beta^2 - \theta\beta + \frac{\xi^2}{4\alpha^4}(1 - 2\alpha\lambda) = 0$$

and define a_1, a_2, b_1, b_2 by

$$a_j = \tfrac{1}{2} b_j - \frac{i}{2}\frac{1}{\sqrt{1-\rho^2}}\left(\frac{\xi}{\alpha^2}(1-\alpha\lambda) - \rho(\theta - 1)\right)$$

$$b_j = 2\beta_j - \theta + 1 \quad .$$

for j =1,2. Then

$$H(t) = K_1 e^{-\beta_1 \alpha t - i x \delta e^{-\alpha t}} M(a_1, b_1, i x e^{-\alpha t})$$

$$+ K_2 e^{-\beta_2 \alpha t - i x \delta e^{-\alpha t}} M(a_2, b_2, i x e^{-\alpha t}).$$

This is valid as long as $b_1 = b$ is not an integer (in which case b_2 is also non-integer, since $b_1 + b_2 = 2$). When b is an integer, one of the functions $M(a,b,z)$ in the equation for H needs to be replaced by the logarithmic solution $U(a,b,z)$ (cf. Abramowitz and Stegun (1977), 13.1.6).

To evaluate the function $F(t)$, we have

$$F(t) = -\frac{2}{\zeta^2} H'(t) / H(t) .$$

Calculating the derivative H' and using the fact that

$$M'(a,b,z) = \frac{a}{b} M(a+1, b+1, z) ,$$

we get the following expression for $F(t)$:

$$F(t) = -\frac{2i\alpha x\delta}{\zeta^2} e^{-\alpha t}$$

$$+ \frac{2\alpha}{\zeta^2} \cdot \sum_{j=1}^{2} K_j e^{-\beta_j \alpha t} \left(\beta_j M(a_j, b_j, ixe^{-\alpha t}) \right.$$

$$\left. + ixe^{-\alpha t} \cdot \frac{a_j}{b_j} M(a_j+1, b_j+1, ixe^{-\alpha t}) \right)$$

$$/ \sum_{j=1}^{2} K_j e^{-\beta_j \alpha t} M(a_j, b_j, ixe^{-\alpha t})$$

The constants K_1, K_2 (obviously, only their ratio is relevant) are chosen to satisfy the boundary condition

$F(0) = 0$.

The function F(t) is real, with a finite limit

$$F(\infty) = \frac{2\alpha\beta}{\xi^2} .$$

Finally, the function G(t) is calculated by integration as

$$G(t) = \bar{F}\left(\frac{1}{\alpha}(1-e^{-\alpha t}) - t\right)$$

$$- \frac{2\gamma\bar{v}}{\xi^2} \cdot \log \frac{H(t)}{H(0)}$$

<u>The Volatility Exposure</u>

From the form of the solution for the bond price

P(t,r,v) = exp(-rD(t)+vF(t)+G(t))

we note that

$$D(t) = -P_r / P$$

$$F(t) = P_v / P .$$

The quantities D,F are thus the rate exposure (i.e., duration) and the exposure to volatility, respectively. Together, the duration and the volatility exposure constitute the risk parameters of a bond. Moreover, since the expected rate of return μ is given by the equation

$$\mu = r + \lambda v D + \eta \xi v F ,$$

the two measures also fully determine the bond expected return.

The explicit form of D,F derived in the previous section applies to a pure discount bond. In order to generalize the concepts to coupon bonds, we will define the duration D^* and the volatility exposure F^* of a fixed-income instrument priced at P^* as

$$D^* = -P_r^* / P^*$$

$$F^* = P_v / P^* .$$

If the bond cashflows are $c_1, c_2, \ldots, c_m$, due at times $t_1, t_2, \ldots, t_m$, then

$$P^* = \sum_{i=1}^{m} c_i P(t_i)$$

and a simple calculation shows that both the duration and the volatility exposure of the bond are determined as weighted averages of those corresponding to the individual cashflows,

$$D^* = \sum_{i=1}^{m} w_i D(t_i)$$

$$F^* = \sum_{i=1}^{m} w_i F(t_i) .$$

The weights are the present values of the payments,

$$w_i = c_i P(t_i) / P^* , \quad i = 1, 2, \cdots, n .$$

The same principle applies to portfolios of fixed-income instruments: both risk measures combine linearly in the market values of the portfolio components.

The dynamics of a discount bond price can be written as

$$\frac{dP}{P} = k\,dt - D\,dr + F\,dv$$

where

$$k = r + (\alpha\bar{r} - \alpha r + \lambda v)D - (\gamma\bar{v} - (\gamma + \xi\eta)v)F$$

is a deterministic part of the drift, and Ddr, Fdv are the stochastic elements. It is easy to establish that the same equation holds for any fixed-income instrument or a portfolio,

$$\frac{dP^*}{P^*} = k^* dt - D^* dr + F^* dv .$$

This means that two portfolios will have the same returns over a given period if, and only if, their durations and their volatility exposures are kept matched during that period.

The volatility exposure thus enters as an additional risk measure. In markets described by the short rate as the only

stochastic factor, duration is the sole risk measure, and portfolios with the same duration will exhibit identical price movements. When the variability of the short rate is itself subject to unforeseen changes, duration alone has to be replaced by the dual measure of risk (and expected return) consisting of duration and the volatility exposure.

The two measures will be relevant to both an active and a passive investor. An active investor takes a position with respect to the market based on a forecast, or expectation, of the factors affecting the portfolio return. If the investor expects a rate increase, say, over that expected by the market he will shorten the duration of his portfolio. An investor expecting more volatile times than the current market pricing implies will decrease his volatility exposure.

A passive investor, on the other hand, strives to reduce or eliminate the risk of negative performance against an investment benchmark. This benchmark may be the present value of the liabilities in pension plan funding, or the value of guaranteed income certificates issued by an insurance company, or the return on a market index used to judge the manager's performance. The passive investor will match the duration, and the volatility exposure, of his portfolio to those of the benchmark.

Regardless of whether the investor pursues an active or passive strategy (the distinction is that of a degree, anyway, since it is determined by the extent to which the investor's expectations differ or not differ from those of the market), he will necessitate means of measuring the portfolio exposure to sources of risk. If interest rate volatility is a factor in pricing of fixed-income securities, measurement and control of volatility exposure in addition to rate exposure should lead to an improvement in portfolio management.

References given above to certain papers are cited as follows:

Brennan, M.J., and Schwartz, E.S., "A Continuous Time Approach to the Pricing of Bonds", Journal of Banking and Finance, 3(1979), 133-155.

Cox, J.C., Ingersoll, Jr., J.E., and Ross, S.A., "A Theory of the Term Structure of Interest Rates", Econometrica, 53(1985), 385-407.

Heath, D., Jarrow, R., and Morton, A., "Bond Pricing and the Term Structure of Interest Rates: A New Methodology", working paper, (1988).

Langetieg, T.C., "A Multivariate Model of the Term Strucutre" , Journal of Finance, 35(1980), 71-97.

Ross, S.A., "Return, Risk, and Arbitrage", Risk and Return in Finance, I.Friend and J.L. Bisksler (Ed.), Ballinger Publishing (1977).

Vasicek, O.A., "An Equilibrium Characterization of the Term Structure", Journal of Financial Economics, 5(1977), 177-188.

Thus, what has been described is an improved system for assessment and management of an asset or portfolio of assets.

CLAIMS

What is claimed is:

1. An apparatus for determining the relative risk of a financial instrument to changing interest rate volatility comprising:

   (a) data storage means for storing historical interest rate volatility data;

   (b) calculation means for calculating a volatility exposure value from said historical interest rate volatility data, said calculation means coupled to said data storage means;

   (c) output means for output of said volatility exposure value.

2. The apparatus as recited by claim 1 further comprising:

   (a) means for specifying an expected level of interest rates in the future;

   (b) means for input of data regarding a financial instrument;

   (c) means for calculating an expected return of said financial instrument, said expected return calculated based on said volatility exposure value and said expected level of interest rates.

3. The apparatus as recited by claim 1 wherein said data storage means comprises a disk drive.

4. The apparatus as recited by claim 1 wherein said calculation means comprises a central processing unit.

5. The apparatus as recited by claim 1 wherein said output means comprises a display.

6. An apparatus for assessing the exposure of a financial instrument to interest rate changes comprising:

   (a) means for input of an historical analysis of interest rate volatility;

(b) means for storing said historical data, said means for storing said historical data coupled to receive said historical data from said means for inputt of said historical data;

(c) means for input of data regarding a futures contract;

(d) means for storing said data regarding a futures contract, said means for storing said data regarding a futures contract coupled to receive said data regarding a futures contract from said means for input of said data regarding a futures contract;

(d) means for calculating a volatility exposure value representative of the percentage change in valuation of said financial instrument resulting from a one unit change in volatility of interest rates, said means for calculating a volatility exposure value coupled to receive said historical data and said data regarding a futures contract; and

(e) output means for displaying said volatility exposure value.

7. The apparatus as recited by claim 6 further comprising:

(a) means for specifying an expected level of interest rates in the future;

(b) means for input of data regarding a financial instrument;

(c) means for calculating an expected return of said financial instrument, said expected return calculated based on said volatility exposure value and said expected level of interest rates.

8. A method for determining the relative risk of a financial instrument to changing interest rate volatility comprising the steps of:

(a) inputting an historical analysis of volatility of interest rates;

(b) calculating a volatility exposure value representative of the percentage change in valuation of said financial instrument resulting from a one unit change in the volatility of interest rates, said step of calculating said volatility exposure value V based on said historical analysis of volatility;

(b) outputting said volatility exposure value.

9. A method for assessing the exposure of a financial instrument to interest rate changes comprising the steps of:

(a) calculating a volatility exposure value representative of the percentage change in valuation of said financial instrument resulting from a one unit change in the volatility of interest rates, said step of calculating said volatility exposure value comprising the steps of:

(i) forming an expectation E1 as to the expected level of volatility in interest rates during period P;

(ii) applying said expectation E1 as a first parameter P1 to a formula F;

(v) said formula F providing, as a result, said volatility exposure value V;

(b) evaluating the relative risks and potential returns of said financial instrument based on said value V.

10. A method for assessing the exposure of a financial instrument to interest rate changes, as recited by claim 9, wherein said evaluation of the relative risks and potential returns of said financial instrument further comprises the steps of:

(a) forming an expectation E2 as to the expected level of interest rates during said period P;

(b) applying said expectation E1 as a second parameter P2 to a formula F.

11. The method as recited by claim 9 wherein said financial instrument is a portfolio of financial devices.

12. The method as recited by claim 9 wherein said financial instrument is a portfolio of fixed income investments.

13. The method as recited by claim 9 wherein said financial instrument is an individual asset.

14. The method as recited by claim 9 further comprising the step of calculating a convexity value.

15. A method of assessing the relative risks and potential returns of a financial instrument comprising the steps of:

(a) calculating a duration value;

(b) calculating a volatility exposure value, said volatility exposure value being calculated as a stochastic factor representative of the percentage change in valuation of said financial instrument resulting from a one unit change in volatility of interest rates;

(c) applying said duration value and said volatility exposure value to calculate an overall expected percentage change in valuation of said financial instrument resulting from a one unit change in volatility of interest rates.

16. The method as recited by claim 15 further comprising the step of calculating a convexity value.

17. An improved method for evaluating risks inherent in a financial portfolio where said financial portfolio has a duration D, said improvement comprising the steps of:

(a) calculating a volatility exposure value V, said volatility exposure value V being calculated as a stochastic factor representative of the percentage change in valuation of said portfolio resulting from a one unit change in volatility of interest rates;

(b) applying said duration D and said volatility exposure value V to calculate an overall expected percentage change in valuation of said financial instrument resulting from a one unit change in volatility of interest rates.

18. An improved method for evaluating expected returns of a financial instrument comprising the steps:

(a) calculating a volatility exposure value, said volatility exposure value being calculated utilizing volatility of interest rates as a stochastic factor; said volatility exposure value representative of a percentage change in valuation of said instrument resulting from a one unit change in volatility of interest rates;

(b) forming an expectation as to changes in volatility of interest rates during a holding period for said instrument;

(c) calculating the difference between the current level of volatility of interest rates and the expected level of volatility of interest rates at the end of said holding period; and

(d) multiplying said difference by said volatility exposure value.

19. A method for determining efficient portfolios of financial instruments based on interest rate level and volatility forecasts comprising the steps of:

(a) calculating an implied market volatility level;

(b) estimating expected interest rate levels at given points in time;

(c) specifying maximum and minimum holdings for each of a plurality of financial instrument classes;

(d) analyzing said financial instruments based on said implied market volatility and said estimated expected interest rate levels; and

(e) providing an efficient portfolio based on said specified maximums and minimums and said analysis.

20. The method of claim 19 wherein said step of calculating said implied market volatility comprises the steps of:

(a) specifying a risk-free interest rate;

(b) specifying a price of a futures contract;

(c) specifying a duration of said futures contract;

(d) specifying data on options for said futures contract; and

(e) calculating said implied market volatility based on said risk-free interest rate, said futures contract and said option.

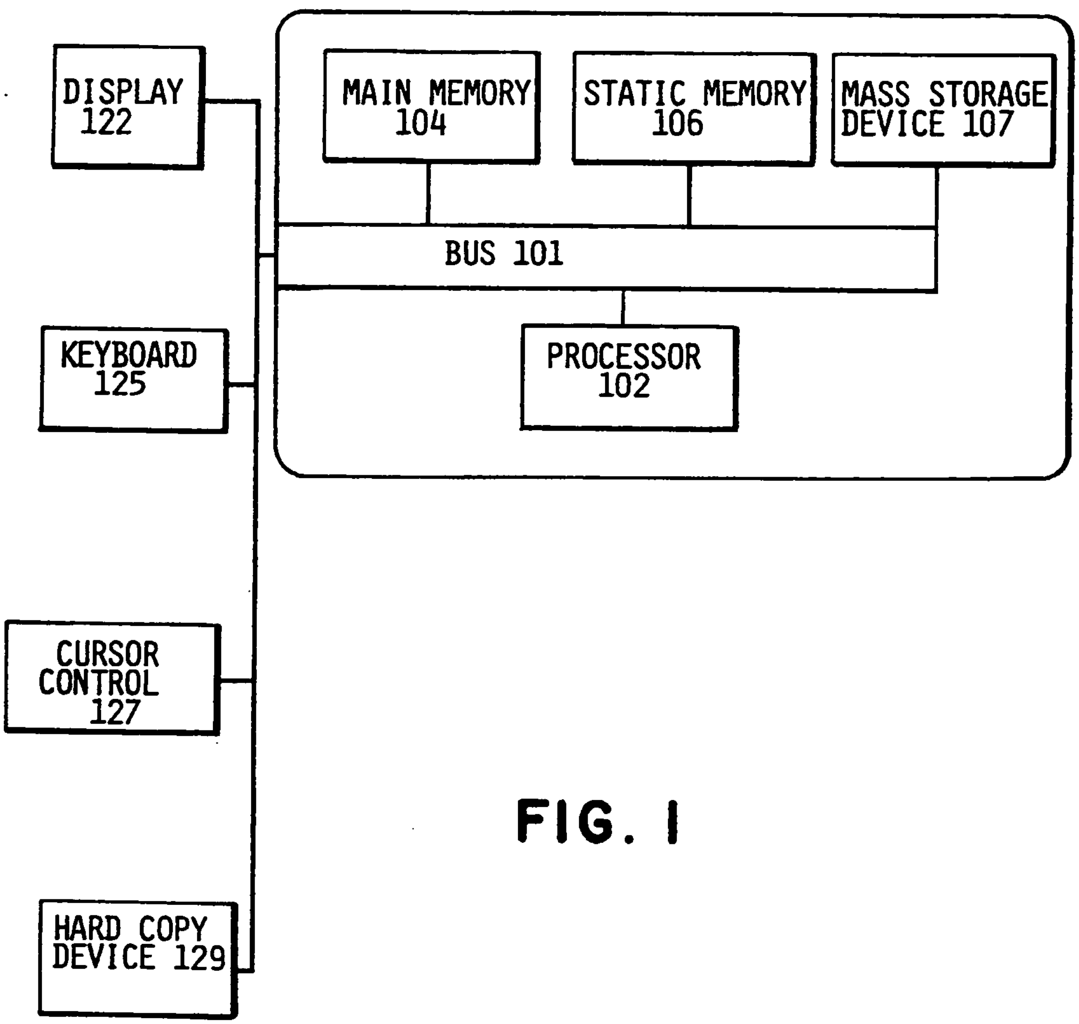

DISPLAY 122
KEYBOARD 125
CURSOR CONTROL 127
HARD COPY DEVICE 129
MAIN MEMORY 104
STATIC MEMORY 106
MASS STORAGE DEVICE 107
BUS 101
PROCESSOR 102

FIG. 1

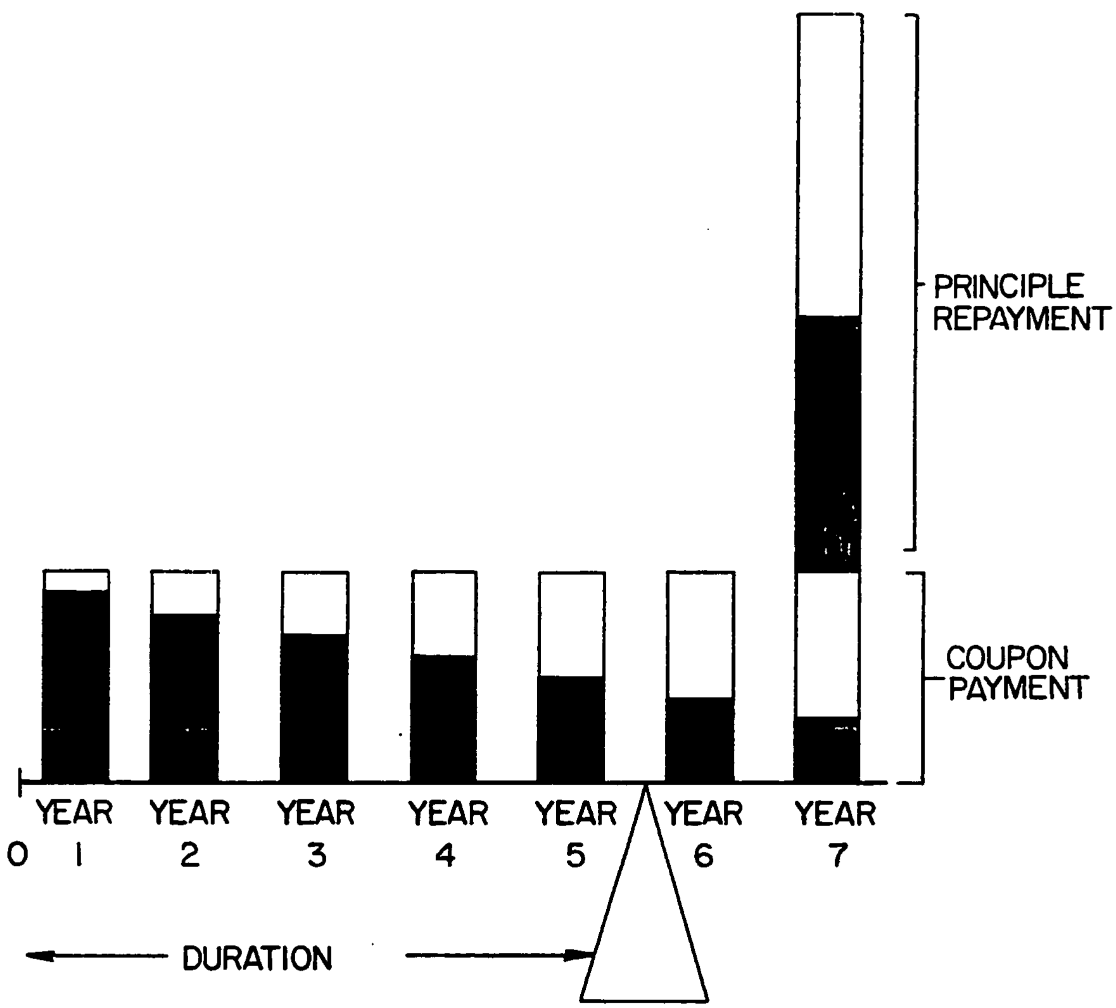

PRINCIPLE
REPAYMENT
COUPON
PAYMENT
YEAR 1
YEAR 2
YEAR 3
YEAR 4
YEAR 5
YEAR 6
YEAR 7
0
DURATION

FIG. 2(a)

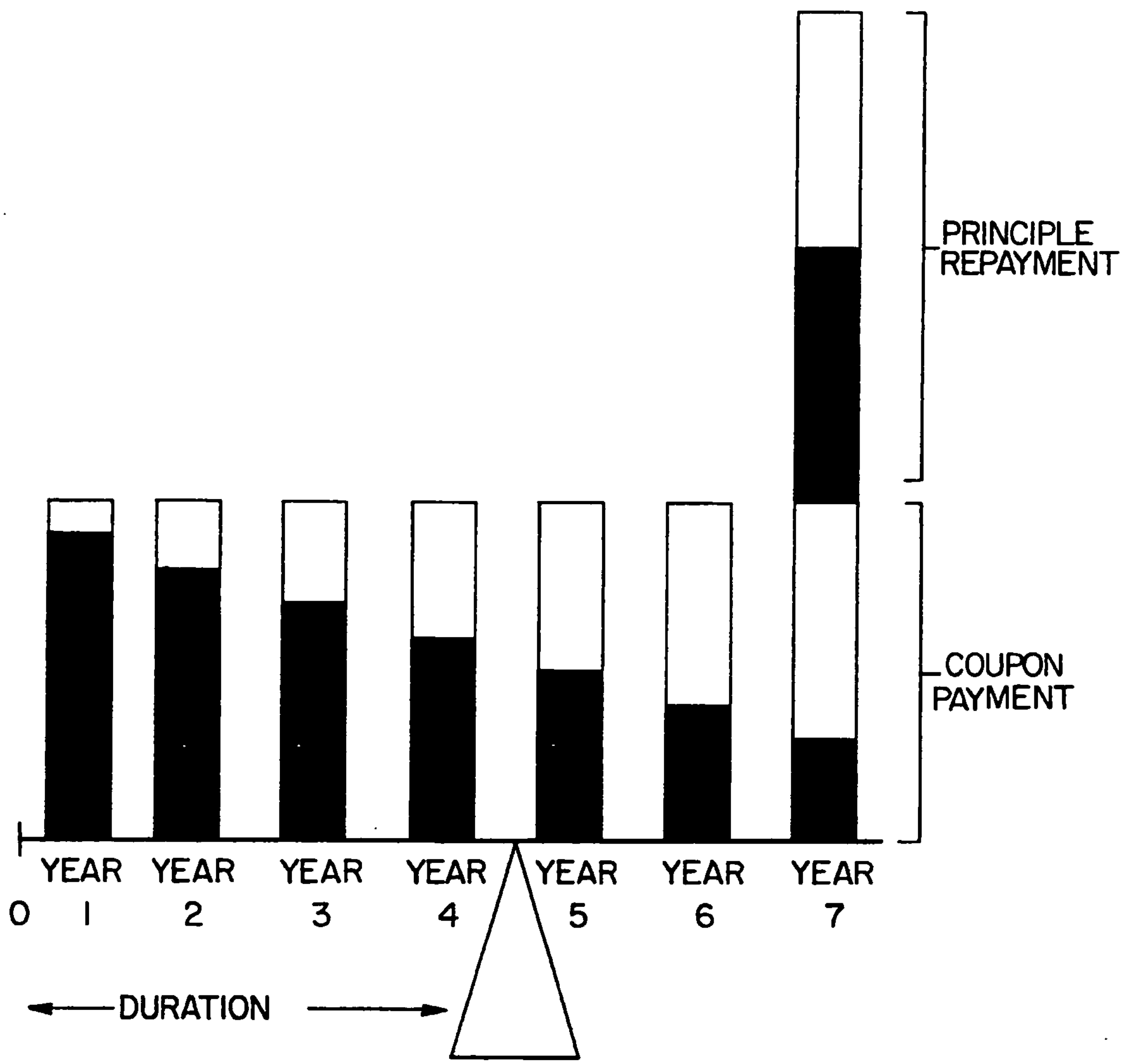

PRINCIPLE REPAYMENT
COUPON PAYMENT
YEAR 1
YEAR 2
YEAR 3
YEAR 4
YEAR 5
YEAR 6
YEAR 7
0
DURATION

FIG. 2(b)

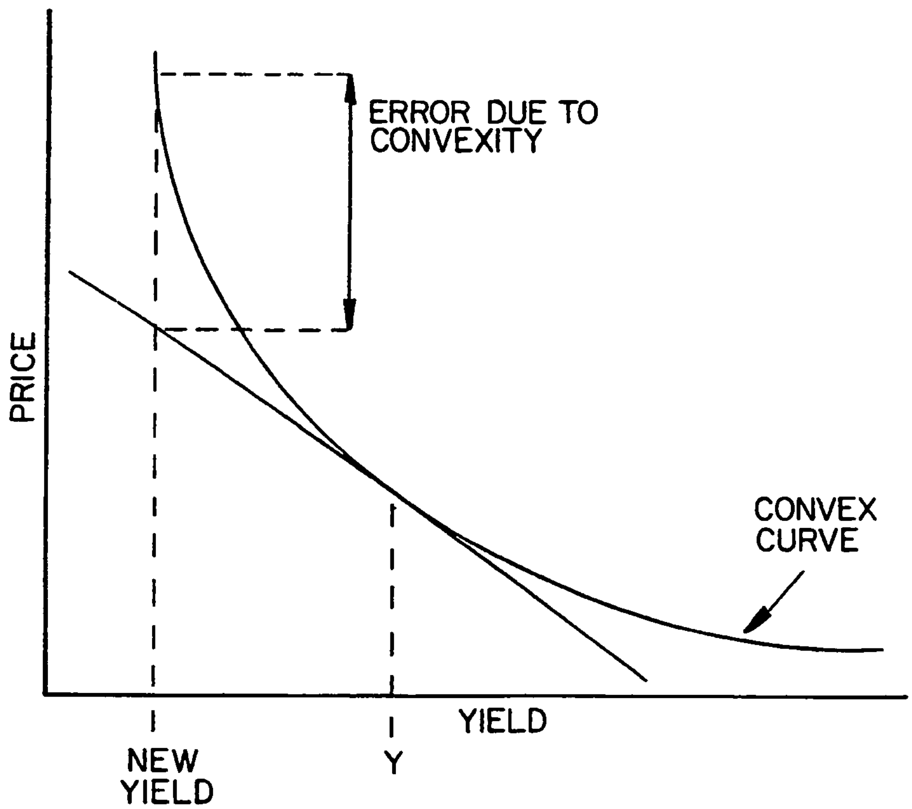


FIG. 3

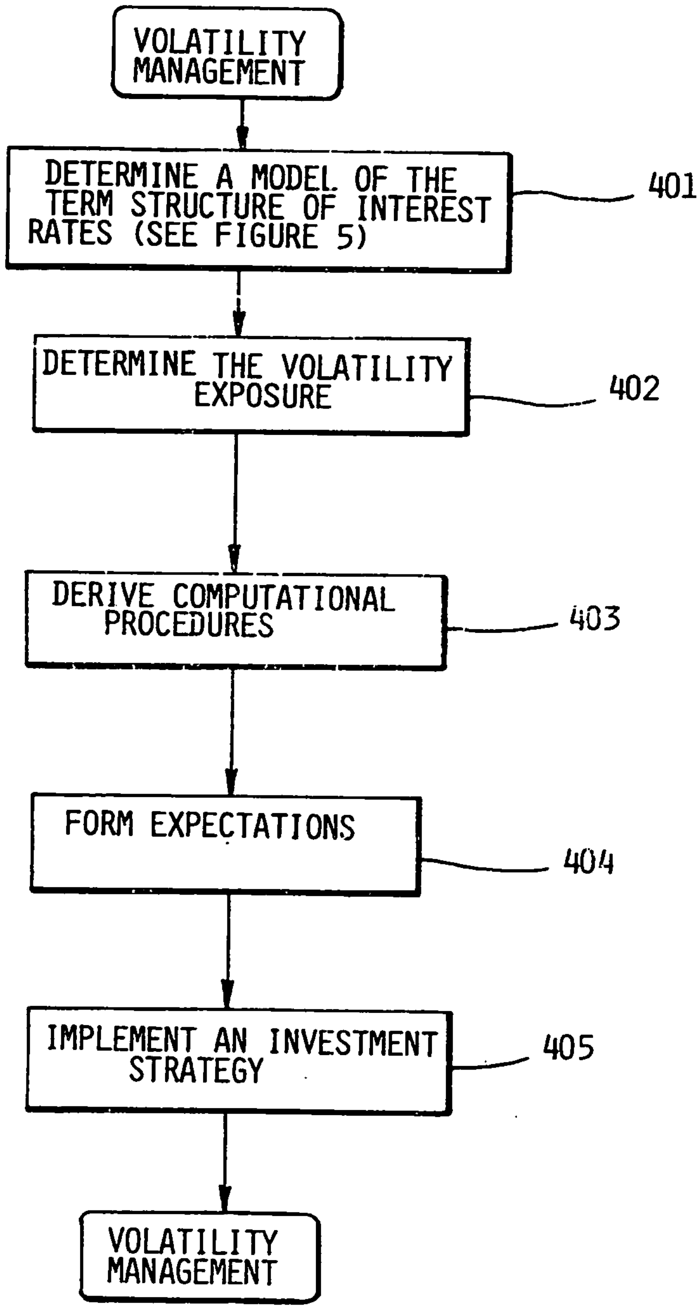

VOLATILITY MANAGEMENT
DETERMINE A MODEL OF THE TERM STRUCTURE OF INTEREST RATES (SEE FIGURE 5)
401
DETERMINE THE VOLATILITY EXPOSURE
402
DERIVE COMPUTATIONAL PROCEDURES
403
FORM EXPECTATIONS
404
IMPLEMENT AN INVESTMENT STRATEGY
405
VOLATILITY MANAGEMENT

FIG. 4

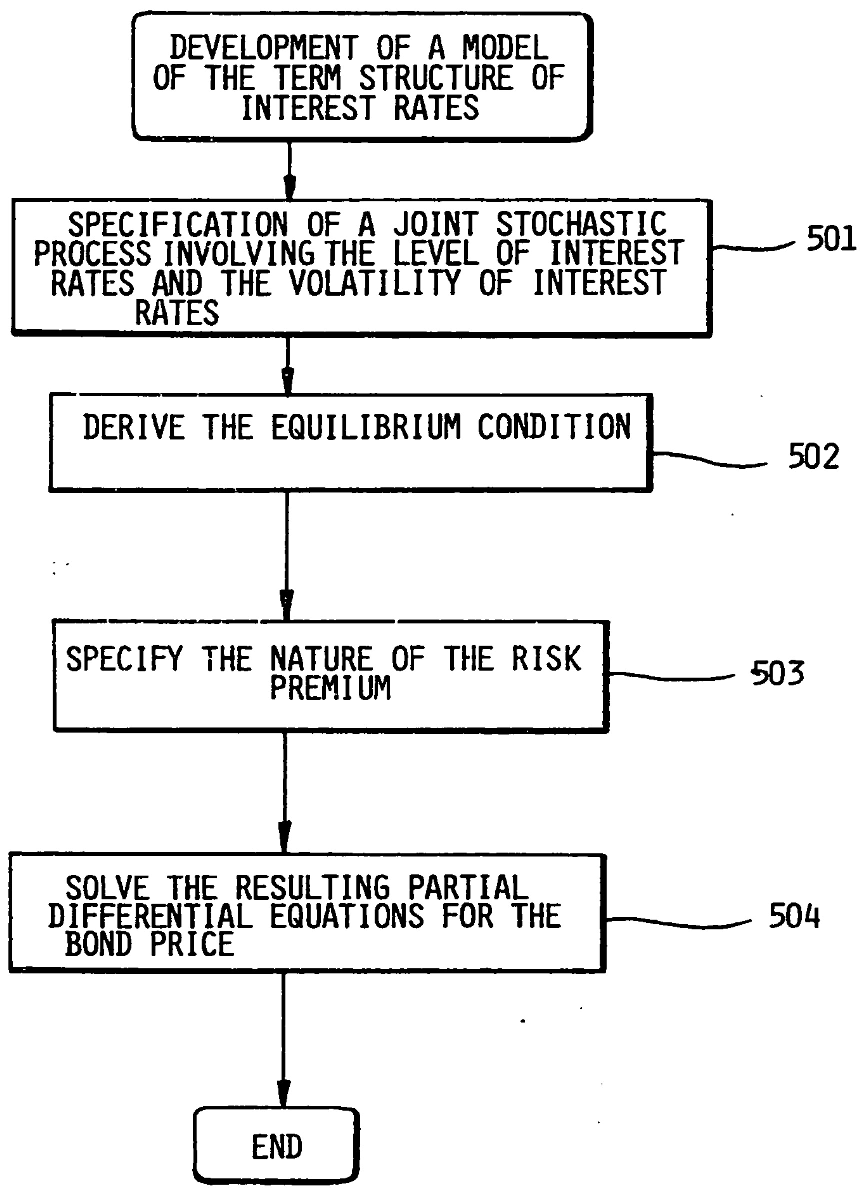

DEVELOPMENT OF A MODEL OF THE TERM STRUCTURE OF INTEREST RATES
SPECIFICATION OF A JOINT STOCHASTIC PROCESS INVOLVING THE LEVEL OF INTEREST RATES AND THE VOLATILITY OF INTEREST RATES
501
DERIVE THE EQUILIBRIUM CONDITION
502
SPECIFY THE NATURE OF THE RISK PREMIUM
503
SOLVE THE RESULTING PARTIAL DIFFERENTIAL EQUATIONS FOR THE BOND PRICE
504
END

FIG. 5

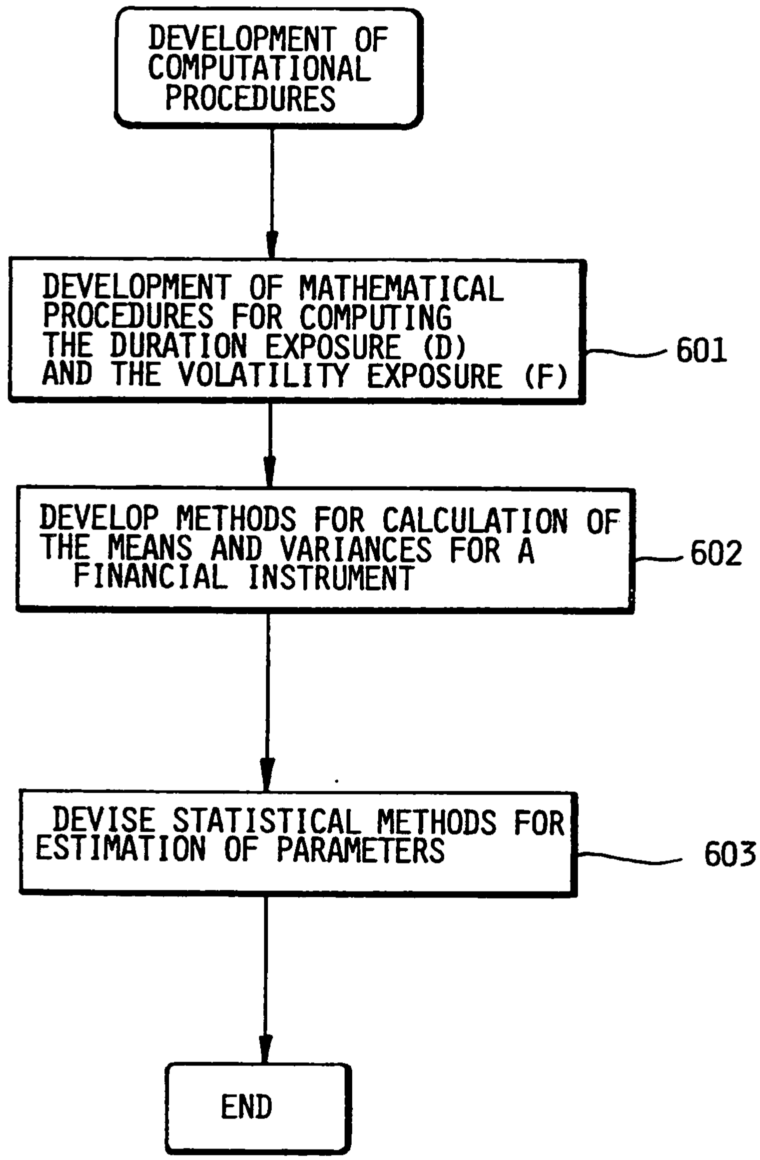

DEVELOPMENT OF COMPUTATIONAL PROCEDURES
DEVELOPMENT OF MATHEMATICAL PROCEDURES FOR COMPUTING THE DURATION EXPOSURE (D) AND THE VOLATILITY EXPOSURE (F)
601
DEVELOP METHODS FOR CALCULATION OF THE MEANS AND VARIANCES FOR A FINANCIAL INSTRUMENT
602
DEVISE STATISTICAL METHODS FOR ESTIMATION OF PARAMETERS
603
END

FIG. 6

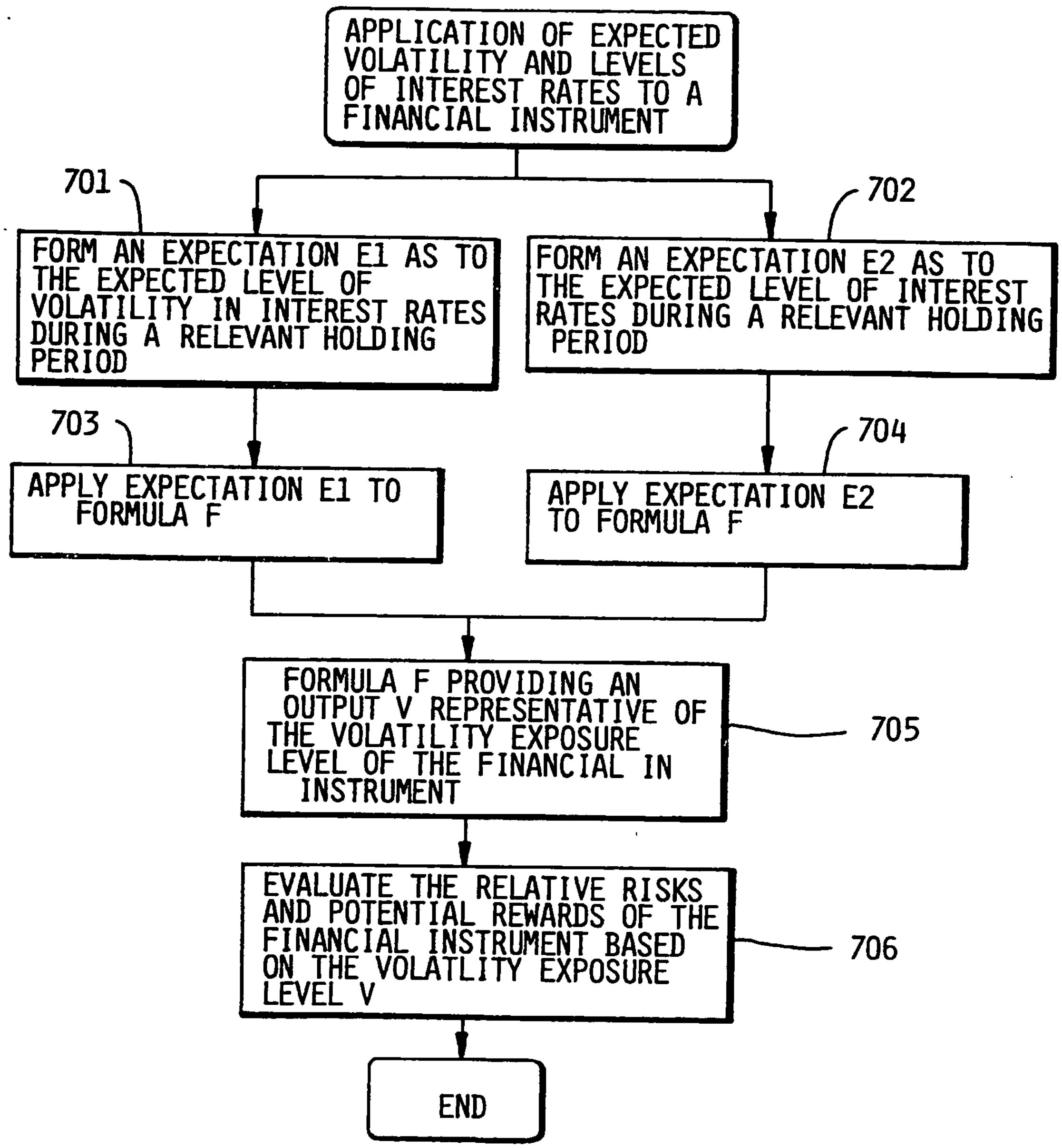

APPLICATION OF EXPECTED VOLATILITY AND LEVELS OF INTEREST RATES TO A FINANCIAL INSTRUMENT
701
FORM AN EXPECTATION E1 AS TO THE EXPECTED LEVEL OF VOLATILITY IN INTEREST RATES DURING A RELEVANT HOLDING PERIOD
702
FORM AN EXPECTATION E2 AS TO THE EXPECTED LEVEL OF INTEREST RATES DURING A RELEVANT HOLDING PERIOD
703
APPLY EXPECTATION E1 TO FORMULA F
704
APPLY EXPECTATION E2 TO FORMULA F
FORMULA F PROVIDING AN OUTPUT V REPRESENTATIVE OF THE VOLATILITY EXPOSURE LEVEL OF THE FINANCIAL IN INSTRUMENT
705
EVALUATE THE RELATIVE RISKS AND POTENTIAL REWARDS OF THE FINANCIAL INSTRUMENT BASED ON THE VOLATLITY EXPOSURE LEVEL V
706
END

FIG. 7

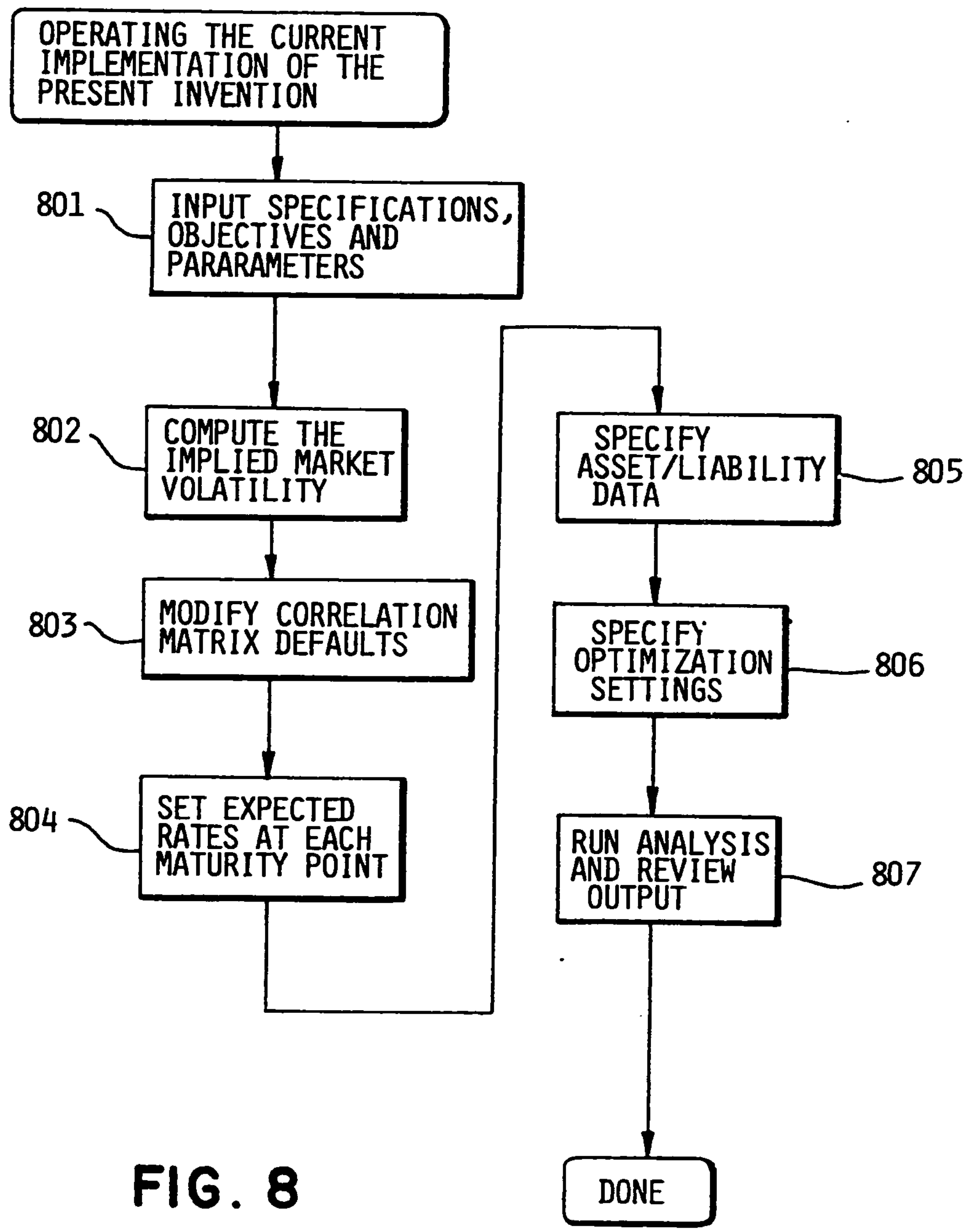

OPERATING THE CURRENT IMPLEMENTATION OF THE PRESENT INVENTION
801
INPUT SPECIFICATIONS, OBJECTIVES AND PARARAMETERS
802
COMPUTE THE IMPLIED MARKET VOLATILITY
803
MODIFY CORRELATION MATRIX DEFAULTS
804
SET EXPECTED RATES AT EACH MATURITY POINT
SPECIFY ASSET/LIABILITY DATA
805
SPECIFY OPTIMIZATION SETTINGS
806
RUN ANALYSIS AND REVIEW OUTPUT
807
DONE

FIG. 8

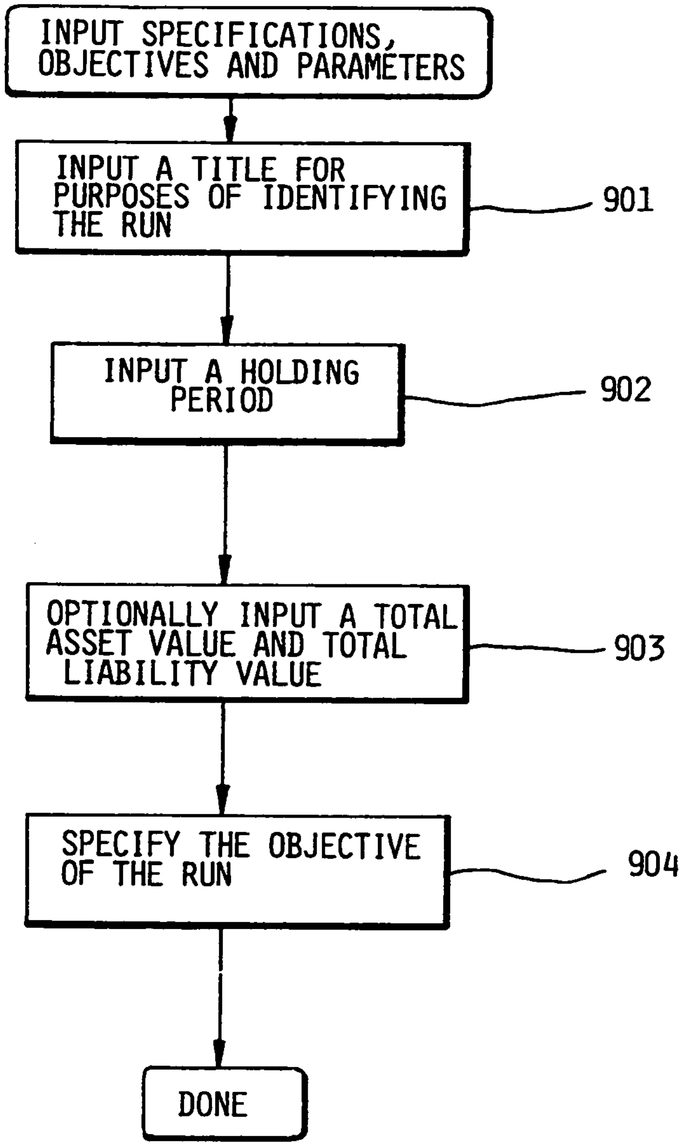

INPUT SPECIFICATIONS, OBJECTIVES AND PARAMETERS
INPUT A TITLE FOR PURPOSES OF IDENTIFYING THE RUN
901
INPUT A HOLDING PERIOD
902
OPTIONALLY INPUT A TOTAL ASSET VALUE AND TOTAL LIABILITY VALUE
903
SPECIFY THE OBJECTIVE OF THE RUN
904
DONE

FIG. 9

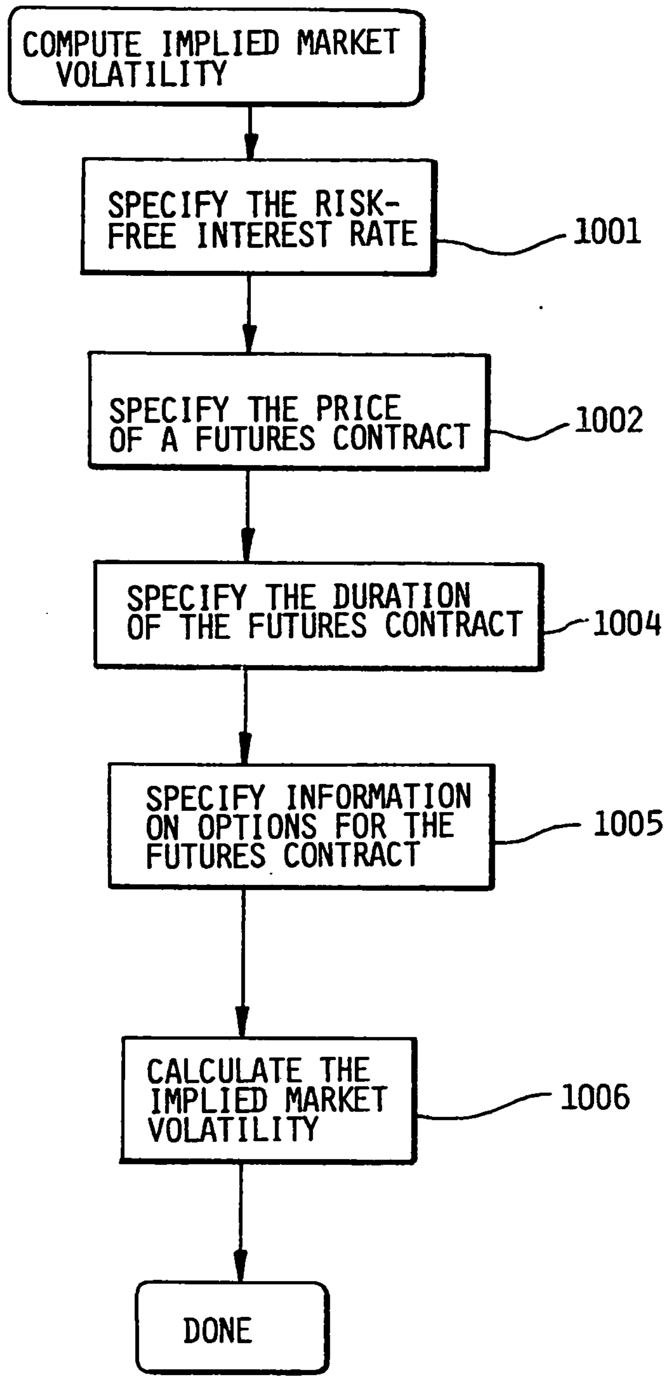
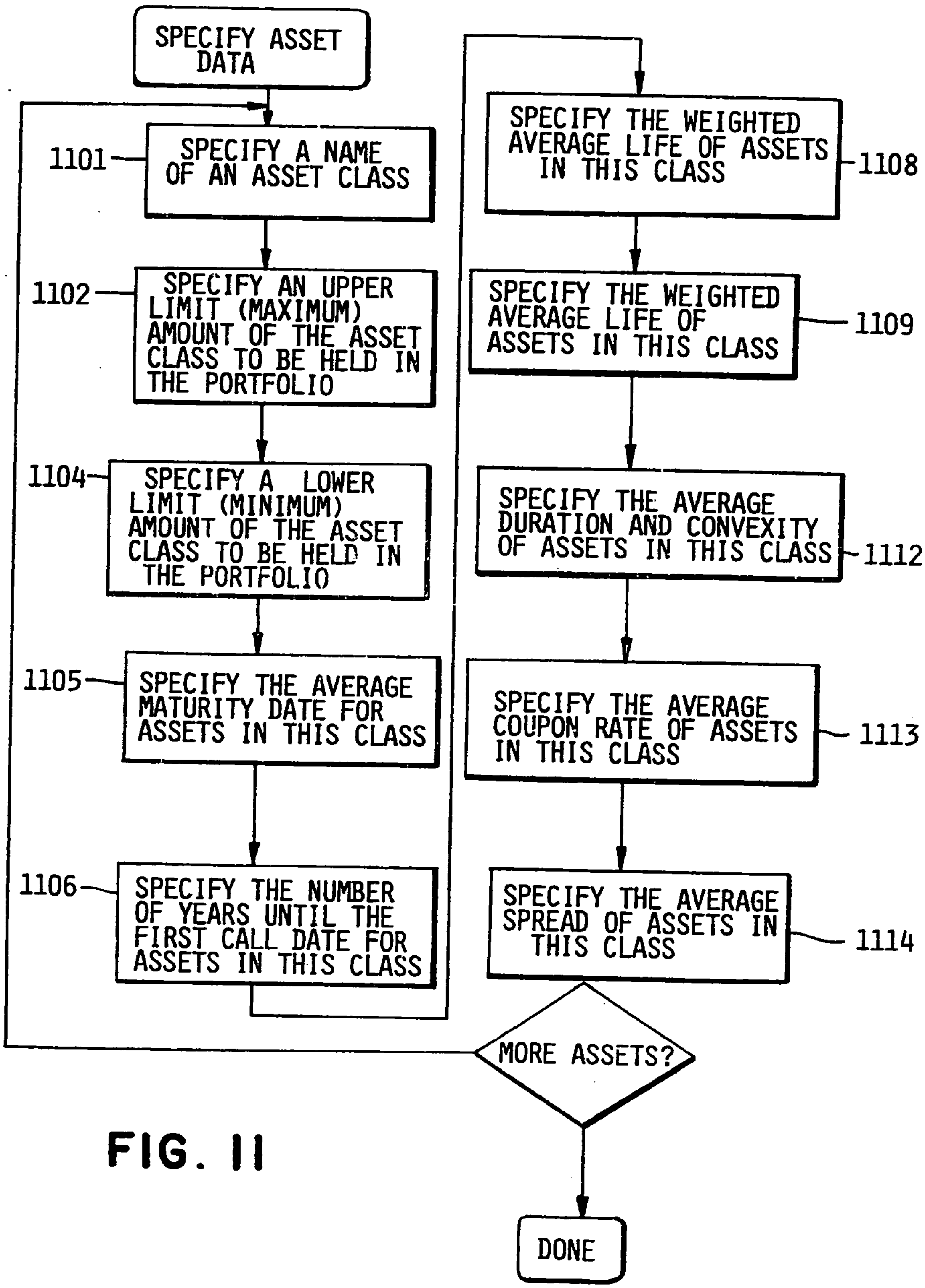

COMPUTE IMPLIED MARKET VOLATILITY
SPECIFY THE RISK-FREE INTEREST RATE
1001
SPECIFY THE PRICE OF A FUTURES CONTRACT
1002
SPECIFY THE DURATION OF THE FUTURES CONTRACT
1004
SPECIFY INFORMATION ON OPTIONS FOR THE FUTURES CONTRACT
1005
CALCULATE THE IMPLIED MARKET VOLATILITY
1006
DONE

FIG. 10



SPECIFY ASSET DATA
1101
SPECIFY A NAME OF AN ASSET CLASS
1102
SPECIFY AN UPPER LIMIT (MAXIMUM) AMOUNT OF THE ASSET CLASS TO BE HELD IN THE PORTFOLIO
1104
SPECIFY A LOWER LIMIT (MINIMUM) AMOUNT OF THE ASSET CLASS TO BE HELD IN THE PORTFOLIO
1105
SPECIFY THE AVERAGE MATURITY DATE FOR ASSETS IN THIS CLASS
1106
SPECIFY THE NUMBER OF YEARS UNTIL THE FIRST CALL DATE FOR ASSETS IN THIS CLASS
SPECIFY THE WEIGHTED AVERAGE LIFE OF ASSETS IN THIS CLASS
1108
SPECIFY THE WEIGHTED AVERAGE LIFE OF ASSETS IN THIS CLASS
1109
SPECIFY THE AVERAGE DURATION AND CONVEXITY OF ASSETS IN THIS CLASS
1112
SPECIFY THE AVERAGE COUPON RATE OF ASSETS IN THIS CLASS
1113
SPECIFY THE AVERAGE SPREAD OF ASSETS IN THIS CLASS
1114
MORE ASSETS?
DONE

FIG. 11

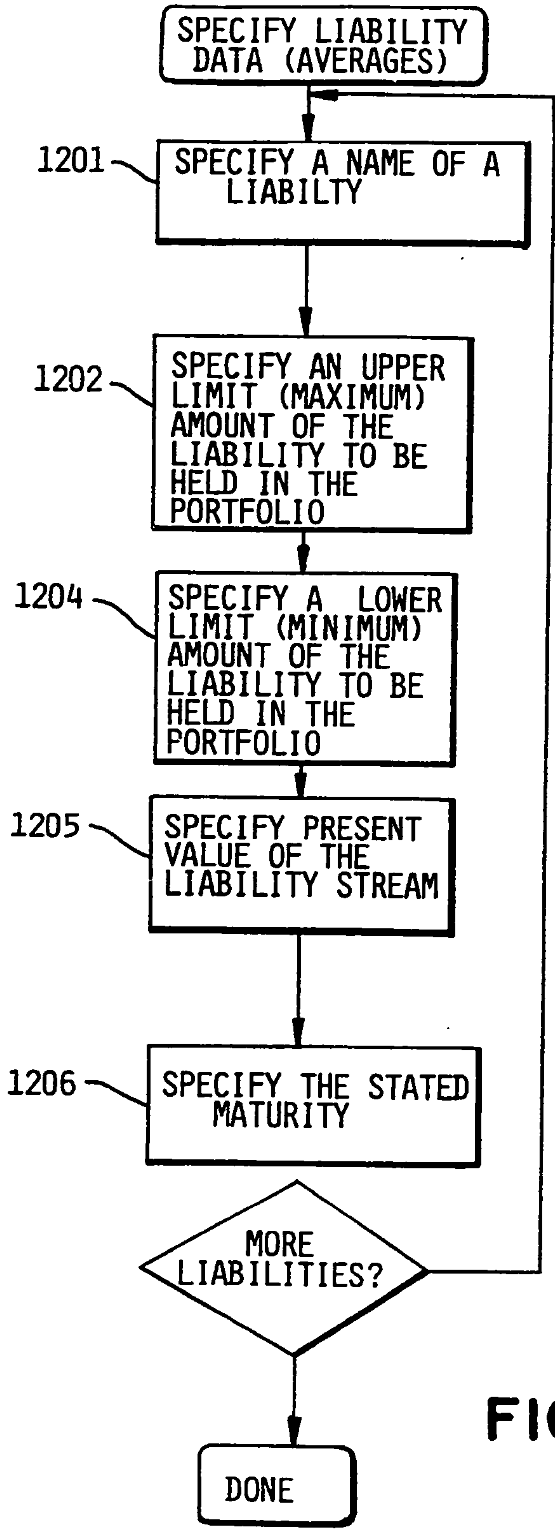

SPECIFY LIABILITY DATA (AVERAGES)
1201
SPECIFY A NAME OF A LIABILTY
1202
SPECIFY AN UPPER LIMIT (MAXIMUM) AMOUNT OF THE LIABILITY TO BE HELD IN THE PORTFOLIO
1204
SPECIFY A LOWER LIMIT (MINIMUM) AMOUNT OF THE LIABILITY TO BE HELD IN THE PORTFOLIO
1205
SPECIFY PRESENT VALUE OF THE LIABILITY STREAM
1206
SPECIFY THE STATED MATURITY
MORE LIABILITIES?
DONE

FIG. 12(a)

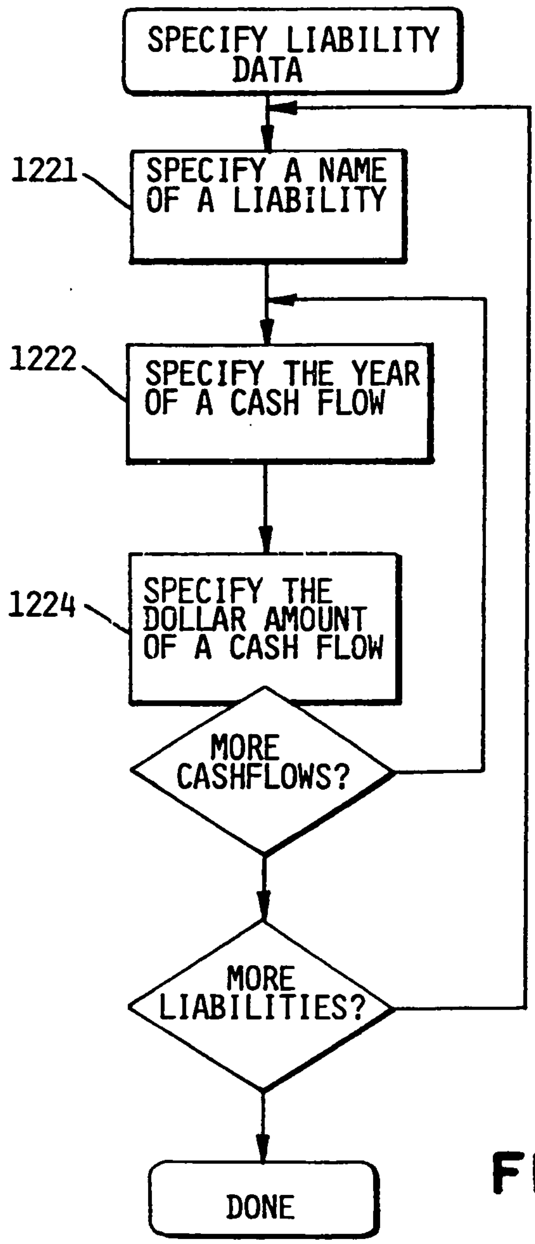

SPECIFY LIABILITY DATA
1221
SPECIFY A NAME OF A LIABILITY
1222
SPECIFY THE YEAR OF A CASH FLOW
1224
SPECIFY THE DOLLAR AMOUNT OF A CASH FLOW
MORE CASHFLOWS?
MORE LIABILITIES?
DONE

FIG. 12(b)

INTERNATIONAL SEARCH REPORT

International Application No PCT/US 92/01445

I. CLASSIFICATION OF SUBJECT MATTER (if several classification symbols apply, indicate all)[6]

According to International Patent Classification (IPC) or to both National Classification and IPC

Int.Cl. 5 G06F15/30

II. FIELDS SEARCHED

Minimum Documentation Searched[7]

Classification System	Classification Symbols
Int.Cl. 5	G06F

Documentation Searched other than Minimum Documentation to the Extent that such Documents are Included in the Fields Searched[8]

III. DOCUMENTS CONSIDERED TO BE RELEVANT[9]

Category °	Citation of Document, [11] with indication, where appropriate, of the relevant passages [12]	Relevant to Claim No.[13]
A	AUSTRALIAN COMPUTER CONFERENCE 1987 8-11 September 1987, Melbourne, Australia. pages 558-577; H. BANNISTER: 'Portfolio Optimisation in the Money Market' see page 569, line 24 - page 570, line 24 see page 572, line 38 - page 573, line 13 ---	1-20

° Special categories of cited documents : [10]

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier document but published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

"&" document member of the same patent family

IV. CERTIFICATION

Date of the Actual Completion of the International Search	Date of Mailing of this International Search Report
27 JULY 1992	17.08.92
International Searching Authority	**Signature of Authorized Officer**
EUROPEAN PATENT OFFICE	BURNE S.R. SRBurne

3